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The budget deficit in an endogenous growth model with bequest and money holdings

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ABSTRACT

By an endogenous growth model with a two-period overlapping generations structure, I examine the existence of a budget deficit in an economy that endogenously grows by investments of firms. The consumers leave bequests to their descendants and hold money as a part of their savings. I use a Barro-type utility function, where people include the utility of their children in their own utility. The main results are as follows. 1) A budget deficit is necessary for full employment under constant prices. 2) Inflation is induced if the actual budget deficit is greater than the value at which full employment is achieved under constant prices. 3) If the actual budget deficit is smaller than the value which is necessary and sufficient for full employment under constant prices, a recession occurs. Therefore, a balanced budget cannot achieve full employment under constant prices. I do not assume that the budget deficit must later be made up by a budget surplus. I use a Barro-type utility function to prove the necessity (or inevitability) of a budget deficit instead of the neutrality of government debt. In the appendix of this paper, I show that if money, as well as goods, are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices.

KEYWORDS

Budget deficit; Growing economy; Endogenous growth model

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ISSN 2972-3272
doi: 10.58567/eal02010005

Received 28 February 2023; Accepted 22 March 2023; Available online 23 March 2023

1. Introduction

In Japan and many other countries, budget deficits or the accumulation of government debt has become a problem, and it is argued that fiscal soundness must be restored. Is this really true? Is it worthwhile to improve fiscal soundness even if seeking a balanced budget will worsen the economy, hinder growth, and generate unemployment? These questions are the starting point of this study. In this paper, using a simple macroeconomic model with microeconomic foundations about consumers and firms, I examine the existence of budget deficits in an endogenously growing economy. In my model, consumers leave bequests to the next generation and hold money as a part of their savings. The significance of government debt and budget deficit and intergenerational burden have been analyzed by J. Tanaka (Jumpei Tanaka, 2010, 2011a, 2011b, 2012). J. Tanaka focuses on the intergenerational economic welfare gap due to the presence or absence of government debt, but his main model does not include economic growth and assumes that all government debt is redeemed by taxes. The interest of this paper lies elsewhere. I am interested in proving that I need budget deficits in a growing economy where consumers hold money. In my model, people leave bequests primarily through the capital, but they are also willing to hold money other than capital for reasons such as liquidity. I use a Barro-type utility function, where people include the utility of their children in their own utility, according to Barro (1974) and, mainly, Carmichael (1982) to prove the necessity (or inevitability) of budget deficit instead of the neutrality of government debt. Also, I use an endogenous growth model with a two-period overlapping generations structure, money holding, and bequests of consumers. About the model, I refer to Grossman and Yanagawa (1993) and, mainly, Maebayashi and J. Tanaka(2021), but my model is a simplified version adding money holding of consumers. [1]

In the next section, I will present the model and prove that a budget deficit is necessary to achieve full employment under constant goods prices. I also show that if the actual budget deficit is greater than what is necessary and sufficient for full employment under constant prices, inflation occurs; if the actual budget deficit is less than what is necessary and sufficient for full employment under constant prices, a recession occurs. Therefore, a balanced budget cannot achieve full employment under constant prices. I do not assume that a budget deficit must later be made up by a budget surplus. [2]

This paper is an example of an analysis using a very simple model of the following statement by J. M. Keynes: "Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e., money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e., a central bank) under public control." (Keynes (1936), Chap. 17)

As Equation (11) shows, when the bequest motive is in effect, the growth rate is determined by the discount rate with respect to descendants' utility, the weight of capital in the production function, and a parameter indicating the relationship between capital and labor productivity.

In the appendix, I show that if money, as well as goods, are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices. However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. The difference between the value of money and its production cost is the so-called seigniorage. A moderate seigniorage to contribute to economic growth without inflation is necessary unless the production of money is quite costly.

2. Budget deficit in monetary economy

[1] I also refer to Otaki (2007, 2009, 2015) about an overlapping generations model.

[2] In another paper, Y. Tanaka (2022), I analyze a similar problem in a traditional overlapping generations model with exogenous economic growth according to Diamond (1965).

2.1. Consumers' behavior

I introduce money demand or money holding and bequest by consumers into a simple endogenous growth model with an overlapping generation's structure. Mainly, I refer to Maebayashi and J. Tanaka (2021) about the formulation of the overlapping generations endogenous growth model. The model is its simplified version, adding money holding and bequests by consumers.

Consumers live over two periods, the younger period and the older period. The population of consumers is constant.

The utility of a younger consumer in Period 1 is represented by

$$V_1 = u_1 + \frac{1}{1+\delta}V_2,$$

where,

$$u_1 = \beta \ln c_1^y + \gamma \ln c_2^o + (1-\beta-\gamma)\ln\frac{m_1}{p_1}, 0 < \beta < 1, 0 < \gamma < 1, 0 < \beta + \gamma < 1.$$

c_1^y is his consumption in Period 1, c_2^o is his consumption in Period 2. In Period 2 he belongs to the older generation. m_1 is his money holding or demand for money in Period 1. β, γ and $1-\beta-\gamma$ are the weights of the consumption in Period 1, the consumption in Period 2, and the money holding in the utility of the consumer.

Consumers derive utility from holding money as well as from consumption in youth and old age. Also, consumers derive utility from the consumption of their descendants, the next generation of consumers. V_2 is the utility of the descendants of a younger consumer. δ is the discount rate about the utility of descendants.

Let b_1 and s_1 be the bequest and the savings of a consumer in Period 1, then

$$b_1 = (1+r_1)(s_1 - m_1) + m_1 - p_2 c_2^o = (1+r_1)s_1 - r_1 m_1 - p_2 c_2^o.$$

r_1 is the interest rate or the rate of return on capital received in Period 2. p_2 is the price of goods in Period 2. From this

$$s_1 = \frac{1}{1+r_1}(b_1 + p_2 c_2^o) + \frac{r_1}{1+r_1}m_1 \tag{1}$$

I assume

$$s_1 - m_1 > 0$$

$s_1 - m_1$ is the investment to capital in Period 1.

The budget constraint for younger consumers in Period 1 is

$$p_1 c_1^y + s_1 = (1-\tau)w_1 l_1 + b_0 = (1-\tau)w_1 l_1 + (1+r_0)s_0 - r_0 m_0 - p_1 c_1^o \tag{2}$$

c_1^o is the consumption in the older period of a consumer of the previous generation. b_0 is the bequest left by previous generation consumers which is equally distributed to consumers in Period 1. w_1 is the nominal wage rate, and τ is the tax rate. The tax is paid by employed consumers. r_0, s_0 and m_0 are the values of each variable in Period 0, which is the previous period of Period 1. If there exists unemployment in Period 0, b_0, s_0, c_1^o and m_0 are interpreted as the average values across employed and unemployed consumers. l_1 is the indicator of whether the consumer is employed or not. If he is employed (unemployed), $l_1 = 1$ ($l_1 = 0$).

Equation (2) is rewritten as

$$p_1c_1^y+\frac{1}{1+r_1}p_2c_2^o+\frac{r_1}{1+r_1}m_1=(1-\tau)w_1l_1+b_0-\frac{1}{1+r_1}b_1.$$

The Lagrange function for the consumers is

$$\mathcal{L}=\beta\ln c_1^y+\gamma\ln c_2^o+(1-\beta-\gamma)\ln\frac{m_1}{p_1}+\frac{1}{1+\delta}V_2$$

$$-\lambda\left[p_1c_1^y+\frac{1}{1+r_1}p_2c_2^o+\frac{r_1}{1+r_1}m_1-(1-\tau)w_1l_1-b_0+\frac{1}{1+r_1}b_1\right].$$

The first order conditions for utility maximization are

$$\beta\frac{1}{c_1^y}=\lambda p_1 \tag{3}$$

$$\gamma\frac{1}{c_2^o}=\lambda\frac{1}{1+r_1}p_2 \tag{4}$$

$$(1-\beta-\gamma)\frac{1}{m_1}=\lambda\frac{r_1}{1+r_1} \tag{5}$$

and

$$\frac{1}{1+\delta}\frac{\partial V_2}{\partial b_1}=\frac{\lambda}{1+r_1}=\frac{\beta}{p_1(1+r_1)c_1^y} \tag{6}$$

From the descendant's budget constraint,

$$\frac{\partial V_2}{\partial b_1}=\frac{1}{p_2}\frac{\beta}{c_2^y} \tag{7}$$

By Eqs. (3), (4) and (5), I get

$$p_1c_1^y=\beta\left[(1-\tau)w_1l_1+b_0-\frac{1}{1+r_1}b_1\right]$$

$$p_2c_2^o=\gamma(1+r_1)\left[(1-\tau)w_1l_1+b_0-\frac{1}{1+r_1}b_1\right]$$

and

$$m_1=\frac{1+r_1}{r_1}(1-\beta-\gamma)\left[(1-\tau)w_1l_1+b_0-\frac{1}{1+r_1}b_1\right]$$

From Eq. (1) the savings is

$$s_1=\frac{1}{1+r_1}b_1+(1-\beta)\left[(1-\tau)w_1l_1+b_0-\frac{1}{1+r_1}b_1\right]$$

$$=\frac{\beta}{1+r_1}b_1+(1-\beta)[(1-\tau)w_1l_1+b_0].$$

Let L_1 and L_f be the employment and the population of consumers (or labor) in Period 1. L_f is constant. I assume that up to Period 0, full employment is achieved and the prices are constant. The total bequest and the total money holding in Period 0 are written as

$$B_0 = b_0 L_f$$

and

$$M_0 = m_0 L_f$$

The total consumption of the younger consumers in Period 1 is

$$C_1^y = \frac{1}{p_1}\beta\left[(1-\tau)w_1L_1 + B_0 - \frac{1}{1+r_1}B_1\right]$$

B_1 is the total bequest in Period 1. The total nominal consumption of the younger consumers in Period 1 is

$$p_1C_1^y = \beta\left[(1-\tau)w_1L_1 + B_0 - \frac{1}{1+r_1}B_1\right] \tag{8}$$

The total money holding in Period 1 is

$$M_1 = (1-\beta-\gamma)\frac{1+r_1}{r_1}\left[(1-\tau)w_1L_1 + B_0 - \frac{1}{1+r_1}B_1\right] \tag{9}$$

The total consumption of the younger consumers in Period 2 is

$$C_2^o = \frac{1+r_1}{p_2}\gamma\left[(1-\tau)w_1L_1 + B_0 - \frac{1}{1+r_1}B_1\right]$$

On the other hand, the total consumption of the older consumers in Period 1 is

$$C_1^o = \frac{1+r_0}{p_1}\beta\left[(1-\tau)w_0L_0 + B_{-1} - \frac{1}{1+r_0}B_0\right]$$

B_{-1} is the total bequest in Period -1 which is the previous period of Period 0.
The total savings in Period 1 is

$$S_1 = \frac{\beta}{1+r_1}B_1 + (1-\beta)[(1-\tau)w_1L_1 + B_0]$$

The real value of the capital in Period 2 is

$$K_2 = \frac{1}{p_1}(S_1 - M_1) = \frac{1}{p_1(1+r_1)}(B_1 - M_1 + p_2C_2^o)$$

The savings of the consumers other than the money holding is invested to the capital.
Similarly, for Period 0,

$$K_1 = \frac{1}{p_0}(S_0 - M_0) = \frac{1}{p_0(1+r_0)}(B_0 - M_0 + p_1C_1^o) \tag{10}$$

2.2. Firms' behavior

I assume full employment and constant prices until Period 0. Thus,

$$L_0 = L_f$$

In Period 1, the production function of each firm is

$$Y_1 = K_1^{\alpha}\left(\theta\widetilde{K}_1 L_1\right)^{1-\alpha}, 0 < \alpha < 1.$$

Y_1, K_1 and L_1 are the output, capital input and labor input. θ is a positive constant. It is larger than 1. $\theta\widetilde{K}_1$ is the labor-augmenting productivity. $\widetilde{K}_1$ is the average capital per population, that is, $\widetilde{K}_1 = \frac{K_1}{L_f}$. However, K_1 in this formulation is the capital over all the economy, and $\widetilde{K}_1$ is given for the firms. The number of firms is normalized to one. The profit of a firm is

$$\pi_1 = p_1 K_1^{\alpha}\left(\theta\widetilde{K}_1 L_1\right)^{1-\alpha} - r_1 p_0 K_1 - w_1 L_1$$

$p_0 K_1$ is the nominal amount of the capital at the time the investment is made, Period 0. K_1 is the amount of capital in Period 1, but the investment in it takes place one period earlier, in Period 0, so p_0, not p_1, is the price of that capital. The first order conditions for profit maximization are

$$p_1 \alpha K_1^{\alpha-1}\left(\theta\widetilde{K}_1 L_1\right)^{1-\alpha} = r_1 p_0$$

and

$$p_1 \theta\widetilde{K}_1 (1-\alpha) K_1^{\alpha}\left(\theta\widetilde{K}_1 L_1\right)^{-\alpha} = w_1$$

In the equilibrium $\widetilde{K}_1 = \frac{K_1}{L_f}$. For simplicity I assume $L_f = 1$, and $0 < L_1 \leqq 1$. Thus, $\widetilde{K}_1 = K_1$, and

$$r_1 p_0 = p_1 \alpha K_1^{\alpha-1}(\theta K_1 L_1)^{1-\alpha} = p_1 \alpha (\theta L_1)^{1-\alpha}$$

or

$$r_1 = \frac{p_1}{p_0}\alpha(\theta L_1)^{1-\alpha}$$

and

$$w_1 = p_1 \theta K_1 (1-\alpha) K_1^{\alpha}(\theta K_1 L_1)^{-\alpha} = p_1 (1-\alpha)\theta^{1-\alpha} L_1^{-\alpha} K_1$$

The wage rate is proportional to the capital input. From them, I obtain

$$p_0 r_1 K_1 = p_1 \alpha (\theta L_1)^{1-\alpha} K_1$$

$$w_1 L_1 = p_1 (1-\alpha)(\theta L_1)^{1-\alpha} K_1$$

$$p_0 r_1 K_1 + w_1 L_1 = p_1 (\theta L_1)^{1-\alpha} K_1$$

and

$$p_1 Y_1 = p_1 K_1^{\alpha}(\theta K_1 L_1)^{1-\alpha} = p_1 (\theta L_1)^{1-\alpha} K_1 = p_0 r_1 K_1 + w_1 L_1$$

The same procedure is used to obtain the values of the variables in Period 0. Since I assume constant prices and full employment until Period 0,

$$r_0 = \alpha\theta^{1-\alpha}, w_0 = p_0(1-\alpha)\theta^{1-\alpha}K_0$$

2.3. Steady state and budget deficit

I consider the steady state with constant prices, constant growth rate, and full employment. In the steady state, the consumption, money holding and the bequest per capita and the capital grows at the constant rate. Let

$$1+g = \frac{K_2}{K_1}$$

g is the growth rate, which is assumed to be positive. Then, I have

$$p_0 = p_1 = p_2, r_1 = r_0, w_1 = (1+g)w_0, C_1^y = (1+g)C_0^y, C_2^o = (1+g)C_1^o,$$

$$M_1 = (1+g)M_0, B_1 = (1+g)B_0 = (1+g)^2 B_{-1},$$

and so on. From Eqs. (6) and (7), in the steady state, I need

$$1+g = \frac{1+r_1}{1+\delta} = \frac{1}{1+\delta}\alpha\theta^{1-\alpha} \tag{11}$$

This means that if the discount rate about the utility of the descendants δ is positive, in a steady-state equilibrium with operative bequests, the interest rate (the rate of return on capital) should be larger than the growth rate, and that the growth rate is determined by the values of the parameters δ, α and θ, that is, the growth rate is determined by the discount rate with respect to descendants' utility, the weight of capital in the production function, and a parameter indicating the relationship between capital and labor productivity.

Since δ is positive and $0 < \alpha < 1$, θ must take a reasonably large value.

Let G_1 be the fiscal spending in Period 1. The market equilibrium condition in the steady state is

$$p_1C_1^y + p_1C_1^o + p_1(K_2 - K_1) + G_1 = p_1Y_1 = p_0r_1K_1 + w_1L_f$$

The left-hand side is the total demand, and the right-hand side is the total supply. $p_1(K_2 - K_1)$ represents the cost required to increase the real value of the capital from K_1 to K_2. Since, by Eqs. (8) and (10)

$$p_1C_1^y = \beta\left[(1-\tau)w_1L_f + B_0 - \frac{1}{1+r_1}B_1\right]$$

$$p_1C_1^o = p_0(1+r_0)K_1 - B_0 + M_0$$

I have

$$\beta\left[(1-\tau)w_1L_f + B_0 - \frac{1}{1+r_1}B_1\right] + p_0(1+r_0)K_1 - B_0 + M_0 + p_1(K_2 - K_1) + G_1$$

$$= p_0r_1K_1 + w_1L_f.$$

From this

$$\beta\left[(1-\tau)w_1L_f + B_0 - \frac{1}{1+r_1}B_1\right] + p_0(1+r_0)K_1 - B_0 + M_0 + p_1(K_2 - K_1) + G_1 - \tau w_1L_f$$

$$= p_0r_1K_1 + w_1L_f - \tau w_1L_f.$$

Therefore,

$$-\frac{\beta}{1+r_1}B_1+p_0(1+r_0)K_1+M_0+p_1(K_2-K_1)+G_1-\tau w_1 L_f$$

$$=p_0 r_1 K_1+(1-\beta)\left[(1-\tau)w_1 L_f+B_0\right].$$

From $p_1=p_0$ and $r_1=r_0$ in the steady state, it is rewritten as

$$-\frac{\beta}{1+r_1}B_1+M_0+p_1K_2+G_1-\tau w_1 L_f=(1-\beta)\left[(1-\tau)w_1 L_f+B_0\right].$$

Since

$$p_1K_2=\frac{1}{1+r_1}(B_1-M_1+p_2C_2^o),$$

and

$$p_2C_2^o=(1+r_1)\gamma\left[(1-\tau)w_1L_f+B_0-\frac{1}{1+r_1}B_1\right],$$

I have

$$M_0+\frac{1}{1+r_1}(B_1-M_1)+\gamma\left[(1-\tau)w_1L_f+B_0-\frac{1}{1+r_1}B_1\right]+G_1-\tau w_1L_f$$

$$=(1-\beta)\left[(1-\tau)w_1L_f+B_0-\frac{1}{1+r_1}B_1\right]+\frac{1}{1+r_1}B_1.$$

Thus,

$$M_0-\frac{1}{1+r_1}M_1+G_1-\tau w_1L_f=(1-\beta-\gamma)\left[(1-\tau)w_1L_f+B_0-\frac{1}{1+r_1}B_1\right]$$

$$=\frac{r_1}{1+r_1}M_1$$

Finally, I get

$$G_1-\tau w_1L_f=M_1-M_0=gM_0 \tag{12}$$

By Eq. (9), so long as $1-\beta-\gamma>0$ and $g>0$, this is positive. Thus, I have shown the following result.

Proposition 1: *If the economy endogenously grows at the positive rate, and the consumers derive positive utility from holding money, a positive budget deficit is needed to maintain full employment under constant prices.*

2.4. Inflation and recession

If the actual budget deficit is smaller than that in Eq. (12), M_1 is smaller than $(1+g)M_0$. Since a consumer divides his budget into consumption and bequest, including money holding, his consumption is also smaller than that when Eq. (12) is satisfied. Then, recession occurs. On the other hand, if the actual budget deficit is larger than that in Eq. (12), M_1 is larger than $(1+g)M_0$, and the nominal consumption also increases. However, under full employment, production can not further increase. Thus, an inflation is triggered.

2.5 Explicit value of the bequest

In this subsection, let us calculate the value of the bequest in the steady state explicitly. Rewrite (12) as

$$G_1 - \tau w_1 L_f = \frac{g}{1+g} M_1 \tag{13}$$

From (9) with $L_1 = L_f$,

$$M_1 = (1-\beta-\gamma)\frac{1+r_1}{r_1}\left[(1-\tau)w_1 L_f + \frac{1}{(1+g)(1+r_1)}(r_1-g)B_1\right].$$

By (11) with $\delta > 0$,

$$r_1 - g > 0.$$

From (13)

$$G_1 - \tau w_1 L_f = \frac{g}{1+g}\frac{1+r_1}{r_1}(1-\beta-\gamma)\left[(1-\tau)w_1 L_f + \frac{1}{(1+g)(1+r_1)}(r_1-g)B_1\right].$$

By (11)

$$G_1 - \tau w_1 L_f = \frac{g}{r_1}(1+\delta)(1-\beta-\gamma)\left[(1-\tau)w_1 L_f + \frac{1}{(1+g)(1+r_1)}(r_1-g)B_1\right].$$

From this

$$\frac{g}{r_1}(1+\delta)(1-\beta-\gamma)\frac{1}{(1+g)(1+r_1)}(r_1-g)B_1 = G_1 - \tau w_1 L_f$$
$$-\frac{g}{r_1}(1+\delta)(1-\beta-\gamma)(1-\tau)w_1 L_f.$$

Let

$$\Phi = \frac{g}{r_1}(1+\delta)(1-\beta-\gamma)\frac{1}{(1+g)(1+r_1)}(r_1-g).$$

Then,

$$B_1 = \frac{1}{\Phi}\left[G_1 - \tau w_1 L_f - \frac{g}{r_1}(1+\delta)(1-\beta-\gamma)(1-\tau)w_1 L_f\right].$$

This is determined by the fiscal expenditure or the budget deficit.

3. Conclusion

In this paper, I have mainly proved that the budget deficit is necessary and inevitable to maintain full employment under constant prices in a growing economy by incorporating consumers' desire to hold money into the overlapping generations endogenous growth model.

This paper does not pursue the causes of involuntary unemployment. However, it is believed that deflation has not occurred to the extent that the real balance effect could realistically eliminate the recession.

Future directions include studying the issues of inflation and recession in more detail and developing a dynamic analysis. Another direction is to study monetary policy by introducing the existence of government bonds. In line with the model in this paper, government bonds are considered to be a substitute for capital, not money. I would also like to examine in depth the implications of equation (11).

Funding Statement

This work received no external funding.

Conflict of interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Acknowledgement

The author deeply appreciates the reviewers' comments, which improved the quality of the paper. The author is responsible for any errors that remain.

Appendix: When money is produced.

In this appendix I suppose that money is produced by capital and labor. The production function is

$$Y_1 + \frac{\widetilde{M}_1}{p_1} = K_1^{\alpha}\left(\theta \widetilde{K}_1 L_1\right)^{1-\alpha}$$

$\widetilde{M}_1$ is the supply of money in Period 1. It is the total cost to produce money in Period 1. The market equilibrium condition for the goods is

$$p_1 c_1^y L_1 + p_1 c_1^o + p_1(K_2 - K_1) + G_1 = p_1 Y_1 - \widetilde{M}_1 = p_0 r_1 K_1 + w_1 L_1 - \widetilde{M}_1$$

The condition for the money market equilibrium is that the supply of money equals an increase in money holding. Therefore,

$$\widetilde{M}_1 = M_1 - M_0$$

Then, under full employment and constant prices, (12) is rewritten as

$$G_1 - \tau w_1 = 0$$

Thus, if money as well as goods are produced by capital and labor, budget deficit is not necessary for full employment under constant prices.

However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. In that case I have

$$\widetilde{M}_1 < M_1 - M_0.$$

Then,

$$M_1 - M_0 - \widetilde{M}_1$$

is the seigniorage. A moderate seigniorage to economic growth without inflation is necessary unless the production of money is quite costly.

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