Economic Analysis Letters

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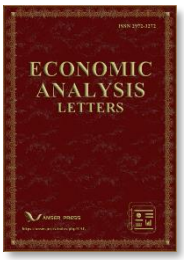

The gap between formalism and empirical science: the example of the non-dictatorship condition

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ABSTRACT

Since the establishment of neoclassical economics in the nineteenth century, there has been a debate in the economics profession over the role played by mathematics. Mathematics can add precision to discussion of real-world empirical problems in economics, but care needs to be taken when formalizing a problem to ensure that errors of translation are not made. Formalism allows one to be sure that a chain of reasoning is correct but applying conclusions back to an empirical science problem is fraught if an error of translation has been made. We illustrate such a difficulty in the context of Arrow's impossibility theorem, specifically the mistranslation of the non-dictatorship condition. The notion of dictatorship entails causality, but causality does not correspond to the usage of the implication sign in mathematics or logic. We use the rules of logic to illustrate that the way that dictatorship is rendered mathematically in the impossibility theorem makes the existence of a dictator (or dictators) not only reasonable but likely.

KEYWORDS

Formalist theory; empirical science; mathematization of economics; impossibility theorem; social welfare function

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ISSN 2972-3272
doi: 10.58567/eal02040005

Received 12 June 2023; Accepted 18 July 2023; Available online 7 August 2023, Version of Record 15 December

1. Introduction

Controversy over the role of mathematics in economics is certainly not new. Pigou (1925: p. 84) credits Marshall with the view that "excessive reliance on this instrument [mathematics] leads us astray in pursuit of intellectual toys, imaginary problems not conforming to the conditions of real life." As one of the founders of neoclassical economics and one competent in mathematics, it is interesting that Marshall already identified this tension in the late nineteenth century.

This concern, that mathematical technique and abstract modelling have been taken too far, was present at the highest levels of the economics profession throughout the twentieth century; see, for example, Hahn (1970), Leontief (1971), and Phelps Brown (1972). Bronfenbrenner (1991: p. 604) puts this view very strongly, "Meanwhile we go our merry ways, debating questions nobody has asked with rigor only a mathematician can appreciate." There are, however, those who are in no doubt that mathematization is the only way for economics to progress. Lucas (1981: p. 276) epitomizes this view when he says that "progress in economic thinking means getting better and better abstract, analogue economic models, not better verbal observations about the world."

Crespo and Tohmé (2017: p.677) describe those who hold the view that the excessive use of mathematics has caused economics to become "detached from reality" as contrarians but take a nuanced view of the defenders of the role of mathematics, splitting them into those who focus on building logically coherent abstract models and those who take a more pragmatic approach encompassing an interaction between empirical data and formal models. Mayer (1993) claims that much of the tension between opposing views could be resolved by drawing a sharper distinction between what he refers to as formalist theory and empirical science theory. Practitioners of the latter type of economics, Mayer argues, have been tempted to apply formalist criteria because of the perceived greater prestige attaching to formalism.

The obvious and moderate response to this controversy is that, in addressing some problems, mathematics has distinct advantages while, for others, it may mislead. However, this begs the question as to how to identify those economic problems for which the application of mathematics is likely to be efficacious. This question is not likely to be resolved theoretically. It is our intention in this paper to focus not on this over-arching question but rather to point out that, even when mathematization is appropriate, one needs to take care to avoid what can be called errors of mistranslation. Such errors, at the interface of economics and mathematics, can render formalism, while formally correct, irrelevant in addressing a real-world (empirical science) problem.

We use Kenneth Arrow's classic paper (Arrow, 1950) to illustrate this point. Arrow's famous impossibility theorem shows that there can be no aggregative device (which he calls a social welfare function) that can take individual preferences and combine them to arrive at collective decisions that satisfy certain reasonable conditions. The argument, which is detailed in the following section, shows specifically that the non-dictatorship condition, which is uncontroversial in everyday language, once formalized, is in fact no longer reasonable.

2. The non-dictatorship condition in Arrow's impossibility theorem

Arrow's impossibility theorem is one of the most famous results in economics or, more correctly, social choice theory. The essential problem of social choice is how the preferences of individuals can be aggregated to arrive at a collective decision. Arrow's result (Arrow, 1950) shows that there can be no aggregation device (a social welfare function) that satisfies certain reasonable principles that we would expect of such a device. The four principles are unrestricted scope, the Pareto principle (or unanimity), non-dictatorship and independence of irrelevant alternatives.

There is no doubt that Arrow considered he was working on an empirical science problem, as a reading of the preface and head note to the reprint of the paper in question will show (Arrow, 1983). The paper itself, after a very

practical introduction on the problems of political decision making, rapidly becomes formal and axiomatic in its approach. Given that the author (Arrow, 1950: p. 328) warns that he is "concerned only with the formal aspects of the foregoing question" and that his result is an impossibility theorem, the formalism need not be of concern *per se*. But, in purporting to address a real-world problem, Arrow enunciates in plain English four reasonable principles, which, when formally expressed, become the basis for his deduction that there is no aggregative device satisfying these principles. Thus, the careful translation of these principles from English to mathematical form becomes critical.

Previously Arrow's impossibility theorem has been attacked in two ways. The first is to argue that one of the reasonable assumptions is not in fact reasonable. The second is to claim that while individually plausible, the set of principles taken together is logically inconsistent. The literature on the subject is voluminous, but is usefully summarised, from a critical perspective, by MacKay (1981). MacKay assesses the plausibility of each principle separately, as well as detailing the argument that only proper subsets of this set of four conditions are consistent in a way that is undoubtedly more accessible than Arrow's formal proof.

Here, our focus is on just one of the reasonable principles. The argument is not that it is unreasonable on face value, but rather that the mistranslation of what is a reasonable proposition in everyday language into one expressed in mathematical (or logical) language makes it unreasonable.

The principle in question is that of non-dictatorship. Arrow (1983: p.71) has claimed this as the least controversial of his conditions. Indeed, in plain English the definition of dictatorship, and its exclusion as a valid means of aggregating individual preferences does seem uncontroversial. Arrow (1950: p.339) puts it this way, "A second form of social choice not of a collective character is the choice by dictatorship. In its pure form this means that social choices are to be based solely on the preferences of one man." Put this way, the idea of a dictatorship is easily grasped and readily dismissed as a way of making collective decisions. Yet, when translated into mathematical form, the words *based solely on the preferences of one man* essentially are rendered with the word *implies*, or rather by an implication sign. Our everyday notion of dictatorship is that the dictator's will is enforced. The dictator's preferences, in this sense, cause a particular state of the world to be chosen over an alternative. But (logical) implication is not causation in this or any other sense. Whatever causality means in economics, including possible econometric versions such as Granger-causality, it does not correspond to the usage of the implication sign in mathematics or logic. The statement A ⇒ B (or "if A then B") is equivalent to A is a subset of B (A ⊂ B). Doing so indicates more clearly that the idea of causality should not even arise since A is not in any sense *causing* B; rather it is simply the case that if A is true, then B is also true.

The "if … then" construction is, of course, central to the axiomatic approach, and there is no wish to deny its usefulness. Why it poses a problem in this instance is best seen by using one simple rule of formal logic. Using a prime (') to denote *not* or the complement of a set, (A ⇒ B) ⇔ (B' ⇒ A'). This rule can be used to transform Arrow's formulation of non-dictatorship. Thus Arrow (1950: p.339) defines a dictatorial social welfare function, with P and P_i meaning *is preferred to* (by society and by individual i, respectively) as follows:

Definition 5: A social welfare function is said to be "dictatorial" if there exists an individual i such that for all x and y, $x\ P_i\ y$ implies $x\ P\ y$ regardless of the orderings of all individuals other than i, where P is the social preference relation corresponding to those orderings.

This definition, as a statement in logic, contains some redundant words, namely regardless of the orderings of all individuals other than i. In plain English, these words have clearly been included merely for emphasis, and could equally well have been rendered by *in all cases*, but a universal statement is not in need of such qualification. Thus, the statement we are dealing with, can be more concisely and more formally expressed as follows, using the existential (∃, there exists) and universal (∀, for all) quantifiers.

$$\exists i\colon \forall (x, y)(xP_i y) \Rightarrow (xPy)$$

But this is equivalent to:

$$\exists i: \forall (x, y)(xPy)' \Rightarrow (xP_i y)'$$

Equivalently, in Arrow's notation, using *R* to mean is preferred or indifferent to, we have:

$$\exists i: \forall (x, y)(yRx) \Rightarrow (yR_i x)$$

If we now re-translate this statement into English, we find that it says:

There exists an individual *i* such that for all *x* and *y*, *y R x* implies *y* R_i *x* or, writing out the meaning of *R* in full, there exists an individual *i* such that for all *x* and *y*, *y* is preferred to or is indifferent to *x* by society implies *y* is preferred to or is indifferent to *x* by individual *i.*

Now, it seems not at all unreasonable that such an individual should exist. Indeed, there is nothing to suggest that there might not be many such individuals in any society. They might well be characterised by a tendency to agree with (*go along with*) whatever choices society has managed to arrive at. In fact, shortly after the publication of Arrow's work, Little (1952: p. 426) offered an example using three individuals and two alternatives where one individual "is deemed dictatorial merely because it happens to be the case that society agrees with him". Little concedes that his example may be contrived and limited but argues that it does raise a paradox. Somewhat confusingly, Little (1952: p. 426) concludes that "it is foolish to accept or reject a set of ethical axioms one at a time". This is surely true, but the difficulty here does lie with the one axiom of non-dictatorship, or rather with the formalised version of it. What we have shown here goes beyond a contrived example to indicate that the idea of dictatorship has been formalised in a way that does not capture its intended meaning.

3. Conclusion

We have shown that there is nothing unreasonable about a dictatorial social welfare function as formalized in the proof of Arrow's impossibility theorem, although all would surely agree that social choice based solely on the preferences of one individual (a dictator) offends reasonable principles of how collective social choices could be made. As MacKay (1981: p. 22) puts it, "Dictatorship is not collective choice at all."

However, much of the argument surrounding Arrow's theorem, as well as the theorem itself, is simply irrelevant in addressing a (real world) empirical science problem. It is so rendered by the error of mistranslation of an everyday notion into mathematical terminology. The problem in hand is then no longer the one that those discussing the problem have in mind. In such a situation, the literature then can take on a life of its own, played out in terms of lemmas, theorems and corollaries, and their proofs, all internally correct, but having no bearing at all on the original question.

Funding Statement

This research received no external funding.

Acknowledgment

Acknowledgments to anonymous referees' comments and editor's effort.

Declaration of Competing Interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Author contributions

Conceptualization: W. Robert J. Alexander; Writing – original draft: W. Robert J. Alexander; Writing – review & editing: W. Robert J. Alexander.

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