Economic Analysis Letters

Homepage: https://www.anserpress.org/journal/eal

Mild Inflation Naturally Prevents Divergence of Debt to GDP Ratio

Yasuhito Tanaka [a, *]

[a] *Faculty of Economics, Doshisha University, Kyoto, Japan*

ABSTRACT

This paper shows mainly the following results. 1) The debt to GDP ratio cannot diverge to infinity, that is, fiscal collapse is impossible. The necessary condition for this result is that the propensity to consume from the asset is positive. 2) The divergence of the debt to GDP ratio is prevented by inflation when the interest rate on government bonds is considerably higher than the real growth rate, and the inflation rate which is sufficient to prevent divergence of the debt to GDP ratio is smaller than the interest rate on government bonds. Only an inflation rate slightly greater than the difference between the interest rate on government bonds and the sum of the real growth rate and the propensity to consume from the asset is required. This inflation is not caused by policy but occurs naturally.

KEYWORDS

Prevention of Debt to GDP Ratio Divergence; Mild Inflation; Budget Deficit

* Corresponding author: Yasuhito Tanaka
E-mail address: ochibocho@gmail.com; yatanaka@mail.doshisha.ac.jp

ISSN 2972-3272
doi: 10.58567/eal03040002

Received 13 September 2024; Accepted 15 November 2024; Available online 24 December 2024; Version of Record 15 December 2024

1. Introduction

Fiscal unsustainability, or the fiscal collapse of public finances, is regarded as an important issue in many modern economies, especially in advanced economies, including Japan. The debt to GDP ratio is often taken up as an indicator of whether or not a country's public finances will fail. A fiscal collapse is considered to occur when the debt to GDP ratio is not limited to a finite value but becomes infinitely large, i.e., when it diverges[1].

This paper will show mainly one result about the debt to GDP ratio, *impossibility of fiscal collapse*. It will show the following theorem. (Theorem 1).

1. The debt to GDP ratio cannot diverge to infinity. The necessary condition for this result is that the propensity to consume from the asset is positive. The consumption from the asset is the key point to our result. The basis for this assumption is that if there are two people who both earn $50,000 per year, for example, and one has $100,000 in assets and the other $1 million, the latter's interest income will be higher and his consumption expenditure will also be higher. The government debt is held as an asset of the people. This is because government debt is the accumulation of the portion of government spending that exceeds tax revenues and the interest income earned on government debt.

If the increase in assets does not increase consumption, that is fine, since there is no longer a concern that inflation will be triggered by an increase in the debt to GDP ratio[2].

2. The divergence of the debt to GDP ratio is prevented by inflation when the interest rate on government bonds is considerably higher than the real growth rate, and the inflation rate which is sufficient to prevent divergence of the debt to GDP ratio is smaller than the interest rate on government bonds. We only require an inflation rate slightly greater than the difference between the interest rate on government bonds and the sum of the real growth rate and the propensity to consume from the asset. This inflation is not caused by policy but occurs naturally. This is due to the fact that when government debt relative to GDP increases, interest payments on it relative to GDP increase, and consumption from the asset relative to GDP also increases.

3. This paper also shows that in order to prevent divergence of the debt to GDP ratio without inflation, taxation on income from asset is necessary.

This paper uses a model based on microeconomic foundations about firms and consumers[3].

Many people believe that government debt issued by the government is debt, a debt owed by the people to the people, and that if the current generation accumulates them, their future descendants will eventually have to repay them. Even in Japan, television and other media report such things as "how much debt is owed per capita". Is this really true? Government spending goes to the non-government private sector, and taxes are collected from the private sector. If the difference between spending and taxes is positive, there is a budget deficit, and the accumulation of this deficit is government debt. The private sector then conversely increases its financial assets as receipts exceed payments. In other words, there are financial assets, not debt, corresponding to the government debt. If the interest rate on government debt (i.e., government bonds) is high, government debt will continue to increase, and its ratio to GDP will increase. Fiscal collapse is often defined as a situation in which the debt to GDP

---

[1] As Blanchard (2022), (2023) notes, many discussions of debt to GDP ratio use simple calculations based on comparisons of primary budget balance (budget deficit excluding interest payments on government bonds), the interest rate, and the growth rate. But is the argument not so simple? Assuming a steady state of full employment, which may or may not include inflation, the size of the budget deficit to achieve this is naturally determined, and the larger the budget deficit is, the higher the inflation rate is. On the other hand, the larger (smaller) the propensity to consume is, the smaller (larger) the budget deficit required to achieve full employment under a constant rate of price increase is.

[2] In our model, the increase in assets does not increase consumption when the interest rate on government bonds is zero, i.e., when money is issued instead of government bonds. Different conditions may arise if the consumers' utility functions differ. Please see (3) and (4) in Section 2, where two different utility functions are compared.

[3] With reference to Otaki (2007), (2009), (2015), Weil (1987), (1988), Maebayashi and Tanaka (2021) and others, we are studying similar problems using an overlapping generations model and a model in which people live infinitely with exogenous growth or endogenous growth. Please see, for example, Tanaka (2023a), Tanaka (2023b), Tanaka (2024a), Tanaka (2024b)and Tanaka (2024c).

ratio does not remain at a finite value but becomes infinitely large. As mentioned above, private financial assets exist in response to government debt, and if the ratio of government debt to GDP is infinitely large, the ratio of private financial assets to GDP should also be infinitely large. This would lead to an increase in consumption. Again, for example, a person with an annual income of $50,000 and assets of $1 million will spend different amounts on consumption than another person with an annual income of $50,000 and assets of $10,000. The accumulation of private financial assets cannot help but increase consumption. If the ratio of private financial assets to GDP is infinitely large, the ratio of consumption to GDP will also be infinitely large. Wait a minute. Since consumption is a part of GDP, its ratio to GDP should be less than 1, so it cannot be infinitely large. Yes, that is why the government will not go bankrupt. What happens if government debt and private financial assets accumulate? Consumption demand increases and prices rise. But only a little. If the nominal growth rate, which is the sum of the real growth rate and the rate of price increases, is only slightly higher than the interest rate on government debt, the government will not go bankrupt. Thus, even with the accumulation of government debt and corresponding private financial assets, a small amount of inflation saves the government from financial ruin. This is not due to government policy. As private financial assets increase, consumption naturally increases and prices naturally rise. If the government debt is owed to foreign countries, the story is different, but so long as the government debt is being serviced domestically, there is nothing to worry about. Let us rid ourselves of the delusion of financial ruin.

In the next section, we present microeconomic foundations about firms and consumers. In Section 3 the main results will be shown. In Section 4 and Section 5 we present simple simulations and an empirical analysis.

Let us start with explanations of microeconomic foundations.

2. Microeconomic foundations for consumers and firms

2.1. Firms

A firm produces one type of good by labor and capital under perfect competition. It determines the employment of labor, the investment of capital and the output of the good to maximize its profit. There exist many firms, but the number of firms is normalized to one. The notations in Period t are as follows.

Y_t: nominal GDP in Period t, $t \geq 0$.

P_t: price of the good in Period t.

y_t: real GDP in Period t, $Y_t = P_t y_t$.

π: inflation rate.

K_t: real capital stock in Period t.

L_t: employment in Period t.

L_f: labor supply in Period t. It is employment under full employment, and is constant.

w_t: wage rate in Period t.

r_t: rate of return on capital in Period t.

γ: real growth rate.

The production function of a firm in Period 0 is

$$y_0 = K_0^{\alpha} L_0^{1-\alpha},\ 0 < \alpha < 1.$$

The profit of a firm is

$$\varphi_0 = P_0 y_0 - w_0 L_0 - r_0 P_0 K_0.$$

The firms determine their employment and capital to maximize their profits. The first order conditions are

$$\frac{\partial \varphi_0}{\partial K_0} = \alpha P_0 K_0^{\alpha-1} L_0^{1-\alpha} - r_0 P_0 = 0,$$

and

$$\frac{\partial \varphi_0}{\partial L_0} = (1-\alpha)P_0 K_0^{\alpha} L_0^{-\alpha} - w_0 = 0.$$

Then,

$$r_0 K_0 = \alpha y_0, \qquad w_0 L_0 = (1-\alpha)Y_0 = (1-\alpha)P_0 y_0.$$

From them

$$r_0 P_0 K_0 + w_0 L_0 = P_0 y_0 = Y_0.$$

It means that the sum of the wage income and the profit of firms is the nominal GDP. For the production in Period $t \geq 1$, we consider labor-augmenting technical progress. The production function in Period t is

$$y_t = K_t^{\alpha}[(1+\gamma)^t L_t]^{1-\alpha}.$$

The profit of a firm is

$$\varphi_t = P_t y_t - w_t L_t - r_t P_t K_t.$$

The first order conditions for profit maximization are

$$\frac{\partial \varphi_t}{\partial K_t} = \alpha P_t K_t^{\alpha-1}[(1+\gamma)^t L_t]^{1-\alpha} - r_t P_t = 0,$$

and

$$\frac{\partial \varphi_t}{\partial L_t} = (1-\alpha)(1+\gamma)^t P_t K_t^{\alpha}[(1+\gamma)^t L_t]^{-\alpha} - w_t = 0.$$

Then,

$$r_t K_t = \alpha y_t, \qquad w_t L_t = (1-\alpha)Y_t.$$

From them

$$r_t P_t K_t + w_t L_t = P_t y_t = Y_t.$$

Under full employment $L_t = L_0 = L_f$. Therefore, with constant rate of return and constant price

$$y_t = (1+\gamma)^t y_0, \qquad K_t = (1+\gamma)^t K_0, \qquad w_t = (1+\pi)^t w_0.$$

Under inflation

$$Y_t = (1+\gamma)^t (1+\pi)^t Y_0, w_t = (1+\gamma)^t (1+\pi)^t w_0.$$

In the steady state growth path the real investment in Period t is

$$K_{t+1} - K_t = \gamma K_t.$$

2.2. Consumers

A consumer determines his/her condumption and holding of government bonds in each period to maximize his/her utility subject to the budget constriant.

The additional notations are as follows.

C_t: nominal consumption in Period t.

G_t: nominal government expenditure in Period t.

S_t: nominal savings of consumers by government bonds at the end of Period $t-1$.

S_{t+1}: nominal savings of consumers by government bonds at the end of Period t.

T_t: nominal income tax.

i: interest rate on government bonds. $i > 0$. It is constant.

Let us consider the utility maximization of consumers whose utility depends on consumption and an increase in savings by holding of government bonds from the previous period. The utility of government bonds is due to liquidity similar to that of money. The investment itself does not produce utility, and the return on investment is included in income (GDP). The utility function is

$$U = \left(\frac{C_t}{P_t}\right)^c \left(\frac{S_{t+1}}{P_t} - \frac{S_t}{P_t}\right)^{1-c}, \qquad 0 < c < 1.$$

c is the propensity to consume. From the previous sub-section,

$$Y_t = w_t L_t + r_t P_t K_t.$$

The budget constraint is

$$S_{t+1} = Y_t - T_t - C_t - P_t(K_{t+1} - K_t) + (1+i)S_t.$$

This means

$$C_t + S_{t+1} - S_t = Y_t - T_t - P_t(K_{t+1} - K_t) + iS_t.$$

Note that S_t and S_{t+1} are, respectively, the savings at the end of Period t and Period $t+1$. It is rewritten as

$$\frac{C_t}{P_t} + \frac{S_{t+1}}{P_t} - \frac{S_t}{P_t} = \frac{Y_t}{P_t} - \frac{T_t}{P_t} - (K_{t+1} - K_t) + i\frac{S_t}{P_t}.$$

Most government bonds are purchased (at least in Japan) by financial institutions, including the central bank and private banks, and ordinary individuals receive payments for government spending in money. However, the portion of government bond interest paid to financial institutions, including private banks, is returned to their shareholders, employees, and bank depositors, so the model assumes that individuals hold government bonds.

The consumers determine C_t and S_{t+1} given S_t. The Lagrange function is

$$\mathcal{L} = \left(\frac{C_t}{P_t}\right)^c \left(\frac{S_{t+1}}{P_t} - \frac{S_t}{P_t}\right)^{1-c} - \frac{1}{P_t}\lambda[C_t + S_{t+1} - S_t - (Y_t - T_t) + P_t(K_{t+1} - K_t) - iS_t].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \frac{C_t}{P_t}} = c\left(\frac{C_t}{P_t}\right)^{c-1} \left(\frac{S_{t+1}}{P_t} - \frac{S_t}{P_t}\right)^{1-c} = \lambda,$$

and

$$\frac{\partial \mathcal{L}}{\partial \frac{S_{t+1}}{P_t}} = (1-c)\left(\frac{C_t}{P_t}\right)^{c} \left(\frac{S_{t+1}}{P_t} - \frac{S_t}{P_t}\right)^{-c} = \lambda.$$

From them

$$C_t : (S_{t+1} - S_t) = c : 1 - c.$$

Let

$$I_t = P_t(K_{t+1} - K_t).$$

It is the nominal investment in Period t. Then,

$$C_t = c(Y_t - T_t - I_t + iS_t) = c(Y_t - T_t - I_t) + ciS_t \tag{1}$$

and

$$S_{t+1} = (1-c)(Y_t - T_t - I_t + iS_t) + S_t = (1-c)(Y_t - T_t - I_t) + (1 + i - ci)S_t \tag{2}$$

ciS_0 is the consumption from the interest income of the asset (government bond), and ci is the propensity to consume from the asset.

Consider another utility function.

$$U = \left(\frac{C_t}{P_t}\right)^c \left(\frac{S_{t+1}}{P_t}\right)^{1-c}, \qquad 0 < c < 1 \tag{3}$$

It depends on the consumption and the savings by holding of government bonds at the end of the period not an increase in the savings. The Lagrange function under the same budget constraint is

$$\mathcal{L} = \left(\frac{C_t}{P_t}\right)^c \left(\frac{S_{t+1}}{P_t}\right)^{1-c} - \frac{1}{P_t}\lambda[C_t + S_{t+1} - S_t - (Y_t - T_t) + I_t - iS_t].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \frac{C_t}{P_t}} = c\left(\frac{C_t}{P_t}\right)^{c-1} \left(\frac{S_{t+1}}{P_t}\right)^{1-c} = \lambda,$$

and

$$\frac{\partial \mathcal{L}}{\partial \frac{S_{t+1}}{P_t}} = (1-c)\left(\frac{C_t}{P_t}\right)^{c} \left(\frac{S_{t+1}}{P_t}\right)^{-c} = \lambda.$$

From them

$$C_t : S_{t+1} = c : 1 - c.$$

Therefore,

$$C_t = c[Y_t - T_t - I_t + (1+i)S_t] = c(Y_t - T_t - I_t) + c(1+i)S_t \tag{4}$$

and

$$S_{t+1} = (1-c)[Y_t - T_t - I_t + (1+i)S_t] = (1-c)(Y_t - T_t - I_t) + (1-c)(1+i)S_t.$$

In this case, the propensity to consume from the asset is $c(1+i)$. (1) and (2) are used in this paper. More generally, a dynamic utility function should be maximized under cross-period budget constraints, where the consumer's utility depends on current and future consumption, government debt holdings, etc. However, since the purpose of this paper is to examine the relationship between divergence/convergence of the debt to GDP ratio and inflation using as basic a model as possible, a static model is used. In the case of dynamic, cross-period utility maximization, both the demand for consumption and the demand for savings may depend on the rate of return on capital, but the essence of the argument in this paper remains the same.

3. The main results

In this section, we present the main results of this paper.

Inductively, it is assumed that

$$S_t = [(1+\gamma+\pi)^{t-1} + (1+\gamma+\pi)^{t-2}(1+i-ci) + \cdots + (1+\gamma+\pi)(1+i-ci)^{t-2} + (1+i-ci)^{t-1}] \\ (1-c)(Y_f - T_0 - I_0) + (1+i-ci)^t S_0 \tag{5}$$

And

$$S_t - S_{t-1} = \\ [(1+\gamma+\pi)^{t-2}(\gamma+\pi) + (1+\gamma+\pi)^{t-3}(1+i-ci)(\gamma+\pi) + \cdots + (1+i-ci)^{t-2}(\gamma+\pi) + (1+i-ci)^{t-1}] \\ (1-c)(Y_f - T_0 - I_0) + (1+i-ci)^t S_0 - (1+i-ci)^{t-1} S_0 \tag{6}$$

S_0 is the savings of consumers by government bonds at the beginning of the world. It may be zero and is nonnegative. The world begins at Period 0. Let Y_0, C_0, I_0, G_0, T_0 be GDP, consumption, investment, government spending, income tax. They are nominal values. Consumers save their assets consisting of government bonds. The GDP is represented by

$$Y_0 = C_0 + I_0 + G_0.$$

From (1)

$$C_0 = c(Y_0 - T_0 - I_0) + ciS_0.$$

Assume full employment. Let Y_f be the GDP at full employment in Period 0. Then,

$$Y_0 = Y_f,$$

and

$$C_0 = c\left(Y_f - T_0 - I_0\right) + ciS_0.$$

The savings of the consumers consisting of government bonds at the end of Period 0 is

$$S_1 = Y_f - T_0 - C_0 - I_0 + (1+i)S_0 = (1-c)\left(Y_f - T_0 - I_0\right) + (1+i-ci)S_0 \tag{7}$$

From this

$$S_1 - S_0 = (1-c)\left(Y_f - T_0 - I_0\right) - (1-c)iS_0 \tag{8}$$

It is carried over to the next period, Period 1. From (7) and (8), (5) and (6) are correct for period 0. The returns to the capital is included in GDP, Y_f. The GDP in Period 0 is

$$Y_f = c(Y_f - T_0 - I_0) + ciS_0 + I_0 + G_0.$$

This means

$$G_0 = (1-c)(Y_f - I_0) - ciS_0 + cT_0.$$

Thus,

$$G_0 - T_0 = (1-c)(Y_f - T_0 - I_0) - ciS_0.$$

This is the budget deficit excluding interest payments on government bonds (the primary budget balance). Then,

$$G_0 - T_0 + (1+i)S_0 = (1-c)(Y_f - T_0 - I_0) + (1+i-ci)S_0 = S_1,$$

and so

$$G_0 - T_0 + iS_0 = S_1 - S_0.$$

This is the budget deficit including interest payments on government bonds in Period 0. It equals the increase in the savings from the beginning of Period 0 to the end of Period 0. The debt to GDP ratio in Period 0 is

$$\frac{S_1}{Y_0} = \frac{(1-c)(Y_f - T_0 - I_0) + (1+i-ci)S_0}{Y_f}.$$

About GDP in Period t,

$$Y_t = C_t + I_t + G_t.$$

The economy nominally grows at the rate of $(1+\pi)(1+\gamma) - 1 = 1 + \pi + \gamma + \pi\gamma$. Since π and γ are small, we neglect $\pi\gamma$, then the nominal growth rate is $\pi + \gamma$. The GDP in Period t is represented as follows;

$$Y_t = (1+\gamma+\pi)^t Y_f = C_t + (1+\gamma+\pi)^t I_0 + G_t \tag{9}$$

The consumption is

$$C_t = c(1+\gamma+\pi)^t\left(Y_f - T_0 - I_0\right) + ciS_t \tag{10}$$

The savings of the consumers consisting of government bonds at the end of Period t is

$$\begin{aligned} S_{t+1} &= (1+\gamma+\pi)^t Y_0 - (1+\gamma+\pi)^t T_0 - C_t - (1+\gamma+\pi)^t I_0 + (1+i)S_t \\ &= (1+\gamma+\pi)^t(1-c)\left(Y_f - T_0 - I_0\right) - ciS_t + (1+i)S_t \\ &= (1+\gamma+\pi)^t(1-c)\left(Y_f - T_0 - I_0\right) + (1+i-ci)S_t \\ &= \left[(1+\gamma+\pi)^t + (1+\gamma+\pi)^{t-1}(1+i-ci) + \cdots + (1+\gamma+\pi)(1+i-ci)^{t-1} + (1+i-ci)^t\right] \\ &\quad (1-c)\left(Y_f - T_0 - I_0\right) + (1+i-ci)^{t+1}S_0 \end{aligned} \tag{11}$$

It is carried over to the next period, Period $t+1$. From (11), (5) is correct. The budget deficit excluding interest payments on government bonds is

$$G_t - T_t = G_t - (1+\gamma+\pi)^t T_0 = (1-c)(1+\gamma+\pi)^t(Y_f - T_0 - I_0) - ciS_t.$$

From this,

$$G_t - T_t + (1+i)S_t = (1-c)(1+\gamma+\pi)^t(Y_f - T_0 - I_0) + (1+i-ci)S_t = S_{t+1}.$$

The budget deficit including interest payments on government bonds is

$$G_t - T_t + iS_t = S_{t+1} - S_t \tag{12}$$

where

$$S_{t+1} - S_t = [(1+\gamma+\pi)^{t-1}(\gamma+\pi) + (1+\gamma+\pi)^{t-2}(1+i-ci)(\gamma+\pi) + \cdots$$
$$+(1+i-ci)^{t-1}(\gamma+\pi) + (1+i-ci)^t](1-c)(Y_f - T_0 - I_0)$$
$$+(1+i-ci)^{t+1}S_0 - (1+i-ci)^t S_0.$$

Thus, the budget deficit including interest payments on government bonds equals the increase in the savings from Period $t-1$ to Period t. From (12), (6) is correct. Given γ, c, i and S_0 (12) means the following result.

Lemma 1 *If the budget deficit in Period t is larger than that in (12) with $\pi = 0$, then an inflation ($\pi > 0$) occurs.*

From (1) and (2), for consumption to be positive, $S_{t+1} - S_t$ must also be positive, which means $G_t - T_t + iS_t>0$, that is, the budget deficit is positive.

If the economy grows, the consumption increases from Period t to $t+1$. Then, $S_{t+1} - S_t > 0$. Therefore, in our model,

Lemma 2 In a growing economy we need positive budget deficit.

The savings (or the asset consisting of government bonds) in Period t is explicitly obtained as follows.

$$S_{t+1} = \sum_{n=0}^{t}\left(\frac{1+i-ci}{1+\gamma+\pi}\right)^n (1+\gamma+\pi)^t(1-c)(Y_f - T_0 - I_0) + (1+i-ci)^{t+1}S_0$$

$$= \frac{1-\left(\frac{1+i-ci}{1+\gamma+\pi}\right)^{t+1}}{1-\frac{1+i-ci}{1+\gamma+\pi}}(1+\gamma+\pi)^t(1-c)(Y_f - T_0 - I_0 + (1+i-ci)^{t+1}S_0$$

$$= \frac{1-\left(\frac{1+i-ci}{1+\gamma+\pi}\right)^{t+1}}{\gamma+\pi-i+ci}(1+\gamma+\pi)^{t+1}(1-c)(Y_f - T_0 - I_0) + (1+i-ci)^{t+1}S_0.$$

The debt to GDP ratio in Period t is

$$\frac{S_{t+1}}{Y_t} = \frac{1-\left(\frac{1+i-ci}{1+\gamma+\pi}\right)^{t+1}}{\gamma+\pi-i+ci}(1+\gamma+\pi)\frac{(1-c)(Y_f - T_0 - I_0)}{Y_f} + \frac{(1+i-ci)^{t+1}}{(1+\gamma+\pi)^t}\frac{S_0}{Y_f}.$$

If

$$\frac{1+i-ci}{1+\gamma+\pi} < 1$$

or

$$i - ci < \gamma + \pi \tag{13}$$

or

$$\pi > i - \gamma - ci$$

then, when $t \to \infty$,

$$\frac{(1+i-ci)^{t+1}}{(1+\gamma+\pi)^t}\frac{S_0}{Y_f} \to 0,$$

and

$$\frac{S_{t+1}}{Y_t} \longrightarrow \left(\frac{1+\gamma+\pi}{\gamma+\pi-i+ci}\right)\frac{(1-c)\left(Y_f-T_0-I_0\right)}{Y_f} \tag{14}$$

Note that ci is the propensity to consume from the asset. (14) is decreasing with respect to c. Also it is decreasing with respect to π since $1+\gamma > \gamma - i + ci$, that is, the higher the rate of price increase, the smaller the limit value of the government debt to GDP ratio. Therefore, it has been shown that

Lemma 3 *The limit value of the debt to GDP ratio, which exists when the interest rate on government bonds is not so high, is decreasing with respect to the propensity to consume and the inflation rate.*

Note that S_0 does not affect the limit value of the debt to GDP ratio.

Our result is a generalized version of the so-called Domar condition (Domar(1944), Yoshino and Miyamoto(2021)) which is based on simple comparison of the interest rate and the nominal growth rate. This condition means that when the interest rate is smaller than the (nominal) growth rate, finances will not collapse. It is stronger than the condition in (13). Consumption from the asset is taken into account in our analysis. It is the key point to our result, and assume full employment with or without inflation.

Now, the following theorem will be shown.

. **Theorem 1**

1. The debt to GDP ratio cannot diverge to infinity.

The necessary condition for this result is that the propensity to consume from the asset is positive.

2. The divergence of the debt to GDP ratio is prevented by inflation when the interest rate on government bonds is high, and the inflation rate which is sufficient to prevent divergence of the debt to GDP ratio is smaller than the interest rate on government bonds. Only an inflation rate slightly greater than the difference between the interest rate on government bonds and the sum of the real growth rate and the propensity to consume from the asset is required.

3. In order to prevent divergence of the debt to GDP ratio without inflation, taxation on income from asset is necessary.

Proof

1. From (9) and (10),

$$(1+\gamma+\pi)^t Y_f = c(1+\gamma+\pi)^t\left(Y_f-T_0-I_0\right) + ciS_t + (1+\gamma+\pi)^t I_0 + G_t \tag{15}$$

Now it is assumed that

$$\frac{1+i-ci}{1+\gamma+\pi} \geq 1,$$

then $\frac{S_t}{Y_{t-1}}$ diverges to infinity when $t \to +\infty$. From (15),

$$1 = c\frac{Y_f-T_0-I_0}{Y_f} + \frac{ci}{1+\gamma+\pi}\frac{S_t}{Y_{t-1}} + \frac{I_0}{Y_f} + \frac{G_t}{Y_t}.$$

$\frac{Y_f-T_0-I_0}{Y_f}$, $\frac{I_0}{Y_f}$ and $\frac{G_t}{Y_t}$ are positive and smaller than one, that is, they are finite. Of course, $1+\gamma+\pi$ and ci are positive and finite. If, when $t \to +\infty$,

$$\frac{S_t}{Y_{t-1}} \longrightarrow +\infty,$$

then, so long as $ci > 0$, that is, the propensity to consume from the asset is positive.

$$\frac{ci}{1+\gamma+\pi}\frac{S_t}{Y_{t-1}} \longrightarrow +\infty.$$

It is a contradiction. Therefore, the debt to GDP ratio cannot diverge to infinity. This means

$$\frac{1+i-ci}{1+\gamma+\pi} < 1$$

and the debt to GDP ratio converges to the value described in (14).

2. If $\pi = 0$, that is, no inflation, and

$$\frac{1+i-ci}{1+\gamma} \geq 1 \tag{16}$$

then, the inflation ($\pi > 0$) prevents the divergence of the debt to GDP ratio. The inflation rate which satisfies

$$\frac{1+i-ci}{1+\gamma+\pi} < 1$$

or

$$\pi > i - \gamma - ci$$

is sufficient to prevent divergence of the debt to GDP ratio[4]. Note that this inflation rate is smaller than the interest rate on government bonds so long as $ci + \gamma > 0$. This inflation is not caused by policy but occurs naturally since as proved in 1 of this theorem divergence of the debt to GDP ratio is impossible. This is due to the fact that when government debt increases relative to GDP, interest payments on it relative to GDP increase, and the consumption from the asset, ciS_t, in (10) and (15) relative to GDP increases.

If inflation causes the initial state where

$$\frac{1+i-ci}{1+\gamma+\pi} \geq 1$$

with a small π or $\pi = 0$ to become a state by an increase in π where

$$\frac{1+i-ci}{1+\gamma+\pi} < 1$$

the government debt at that point can be used as S_0 which is the basis for the subsequent state.

The debt to GDP ratio in Period $t-1$ with constant price ($P_t = 1$ for all t) and $S_0 = 0$ (zero initial government debt) is

$$\frac{S_t}{Y_{t-1}} = \left[\frac{\left(\frac{1+i-ci}{1+\gamma}\right)^t - 1}{i-\gamma-ci}\right]\frac{(1+\gamma)(1-c)(Y_0-T_0-I_0)}{Y_0}.$$

Let $\frac{\tilde{G}_t}{Y_t} < 1$ be an appropriate value of fiscal expenditure to GDP ratio. If

$$\frac{1+i-ci}{1+\gamma} > 1, \qquad or\ \ i > \gamma + ci,$$

$\frac{S_t}{Y_{t-1}}$ grows as much as one wants. On the other hand, $\frac{I_0}{Y_f}$ and $\frac{\tilde{G}_t}{Y_t}$ are smaller than one. Then, there exists t such that

[4] We assume that the central bank will not tighten monetary policy unless prices rise substantially, and that the interest rate on government bonds will remain constant with moderate inflation. It is unrealistic to think that the interest rate will rise to satisfy (16) and that prices will rise in response because interest payments increase, and the interest rate will rise without limit.

$$1-c\frac{Y_f-T_0-I_0}{Y_f}-\frac{ci}{1+\gamma}\frac{S_t}{Y_{t-1}}-\frac{I_0}{Y_f}<\frac{\tilde{G}_t}{Y_t}.$$

In this case, inflationary pressure is exerted. With some inflation rate π

$$1-c\frac{Y_f-T_0-I_0}{Y_f}-\frac{ci}{1+\gamma+\pi}\frac{S_t}{Y_{t-1}}-\frac{I_0}{Y_f}=\frac{\tilde{G}_t}{Y_t} \tag{17}$$

If with this π still

$$i>\pi+\gamma+ci,$$

the debt to GDP ratio further grows. Eventually, there is an inflation rate π^* such that

$$i<\pi^*+\gamma+ci, \qquad or\ \pi^*>i-\gamma-ci,$$

and (17) holds with $\pi=\pi^*$. Then, with this inflation rate the debt to GDP ratio converges to

$$\left(\frac{1+\pi^*+\gamma}{\pi^*+\gamma-i+ci}\right)\frac{(1-c)(Y_0-T_0-I_0)}{Y_f} \tag{18}$$

The magnitude of fiscal spending to GDP ratio may need to be adjusted to achieve full employment under the constant inflation rate π^*.

Even if the debt to GDP ratio does not diverge, consumption from assets may increase, which, when combined with investment and government spending, may lead to excess demand and cause inflation. In such a case, the debt to GDP ratio converges to a value based on the inflation rate expressed in (18).

3. Let τ $(0<\tau<1)$ be the tax rate on the interest payments on government bonds. If

$$\frac{1+(1-\tau)(1-c)i}{1+\gamma}<1$$

is satisfied, the debt to GDP ratio does not diverge without inflation. Since $c>0$, $i>0$ and $\gamma>0$, there exists such a value of τ. (Q.E.D.)

4. Simple simulations

Since the main argument of this paper is a theoretical analysis, a simulation in this section and an empirical analysis in the next section should be considered supplementary.

Let us assume $\gamma=0.01$, $i=0.03$, $c=0.6$, $Y_f=500$, $I_0=100$, $T_0=100$.

Table 1 below show the changes in debt to GDP ratio for π=0.03, π=0.018 and π=0 (constant prices), assuming $S_0=0$. For π=0.03, the debt to GDP ratio converges to 45.71, for π=0.018, it converges to 79.25. On the other hand, when π=0, the debt to GDP ratio does not converge and diverges to infinity.

Table 1. The changes in debt to GDP ratio for π=0.03, π=0.018 and π=0 (constant prices).

π=0.03		π=0.018		π=0	
year	Debt to GDP ratio	year	Debt to GDP ratio	year	Debt to GDP ratio
1	1.23	1	1.23	1	1.24
2	2.43	2	2.45	2	2.48
3	3.59	3	3.64	3	3.72
5	5.83	5	5.98	5	6.21
10	10.92	10	11.51	10	12.49
15	15.36	15	16.62	15	18.82
20	19.23	20	21.34	20	25.22
25	22.61	25	25.71	25	31.69
30	25.56	30	29.75	30	38.21

35	28.13	35	33.48	35	44.81
40	30.37	40	36.93	40	51.47
50	34.04	50	43.08	50	64.98
100	42.73	100	62.74	100	136.72

In the cases where π=0.03 and π=0.018, $i - ci < \gamma + \pi$ is satisfied. On the other hand, when π=0, it is not. As we have shown in Lemma 3, the limit value of the debt to GDP ratio is decreasing with respect to the inflation rate.

Inflation is not good for people, but if rising government debt to GDP ratio is a problem, it may be desirable if divergence of debt – GDP ratio can be prevented by a moderate inflation. In the previous section, we demonstrated that this can be achieved naturally, not by policy.

5. Empirical analysis

Based on the analysis so far, there may be an inverse relationship between the debt to GDP ratio and the price index. In this section, we present a very basic empirical analysis of this relationship. OECD data (https://data.oecd.org) are used. Data on debt to GDP ratios and the price indices for the main countries from 2012 to 2022 are presented in Tables 2 and 3 below. Based on the averages for the countries in the table, regression of the debt to GDP ratio to the price index yields the following result. Let

X: Price index.

Y: Debt to GDP ratio (%).

Then, we obtain

$$Y = 288.07 - 1.719X \ (R^2 = 0.299).$$

t-value is -1.958. It means that the price index is inversely related to the debt to GDP ratio.

Table 2. Debt to GDP (%) ratio from 2012 to 2022.

	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Australia	58.4	55	60.5	64	68.8	66.2	66.8	76.4	91.2	83.1	70.8
Canada	113.7	107.8	108.6	114.4	115.4	111.8	109.8	111.9	146.1	134.1	113.3
France	111.9	112.5	120.2	120.8	123.7	122.9	120.7	123.1	145.5	138	117.3
Germany	89.1	84.5	84.3	80.1	77.3	73.3	70.1	68.7	81.6	79.1	65.4
Italy	135.4	143.2	155.7	156.9	154.6	152.1	146.9	154.2	183.3	171.4	148.5
Japan	226.6	229.7	234.4	233.3	231.4	230.3	234.2	234.8	257	256	254.5
Korea	47.5	47.9	50.7	52.5	51.6	49.4	50.4	52.7	58.9	59.9	57.5
South Africa	53	54.2	58	55.8	60.6	61.5	63.5	68.6	80.8	80.4	74.9
United Kingdom	109	104.7	114.4	114	121.1	120.6	117.4	120.1	152.7	142.6	104.5
United States	132.3	135.8	135.5	136.9	138.8	135.4	137.3	136.1	159.9	148.1	144.2
Average	107.69	107.53	112.23	112.87	114.33	112.35	111.71	114.66	135.7	129.27	115.09

Table 3. Price index from 2012 to 2022.

	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Australia	158	143	135	126	125	131	126	121	118	125	123
Canada	123	121	115	111	105	108	107	110	108	114	117
France	107	110	111	102	100	101	102	95	96	98	92
Germany	100	105	106	98	96	97	100	96	98	100	96
Italy	95	100	102	93	90	90	92	87	87	88	82
Japan	129	106	101	97	112	109	108	113	113	107	92

Korea	75	81	86	86	86	89	89	87	84	86	80
South Africa	62	56	53	52	48	56	56	55	50	56	53
United Kingdom	110	111	119	120	108	102	105	102	104	107	105
United States	99	102	103	114	116	116	115	118	118	115	125
Average	105.8	103.5	103.1	99.9	98.6	99.9	100	98.4	97.6	99.6	96.5

The same regression analysis was performed for Japan and found the following.

$$Y = 282.36 - 0.4075X \ (R^2 = 0.12).$$

t-value is -1.1. An inverse correlation between the debt to GDP ratio and the price index has emerged but is not a strong result.

The arguments in this section are similar to those in the previous section. A higher inflation rate increases the nominal growth rate, so the government debt-to-GDP ratio is not easy to grow, and the limit value at which it converges is smaller.

6. Conclusion

In Japan and elsewhere, there is often concern about the accumulation of government debt and the increase in the debt to GDP ratio. However, the goal of macroeconomic policy is to achieve near full employment while avoiding inflation as much as possible, not to control government debt. This paper showed that an increase in the debt to GDP ratio is not a matter of concern. It is assumed that the budget deficit is financed entirely by the issuance of government bonds, but some or all of the deficit could be financed by money. In that case, fiscal collapse would not occur because interest payments on government debts are low or zero.

There is a strong belief among many that government debt must eventually be repaid through taxes, and that new government spending through the issuance of government bonds will be a burden on future generations. However, the author believes that this may not be the case. Regardless of who purchases the government bonds, government spending increases the financial assets of those who receive the spending, while taxation decreases the financial assets of people. The difference between the two is the budget deficit, and if the budget deficit continues, the financial assets held by people will continue to increase along with government debt. What kind of problems will these accumulated financial assets cause? People’s consumption is considered to depend on accumulated assets as well as income each period. Therefore, if people’s financial assets increase along with government debt, this will increase consumption, leading to higher prices under full employment. Although the ratio of government debt to GDP is often considered more problematic than the total amount of government debt, the ratio of government debt to GDP will decline as prices rise and nominal GDP increases. This paper focused its analysis on that point.

Whenever the government implements any policy, financial resources are always an issue. Increasing taxes reduces people’s disposable income, which reduces consumption and worsens the economy. When the economy is booming or at full employment, new government spending becomes an inflationary factor and tax increases are necessary to control it, but even when the economy is not doing well, the question of financial resources becomes an issue. This is because of the assumption that the government bonds must be repaid. Government spending increases demand, while taxes reduce demand by reducing disposable income. It is the government’s duty to balance the two and to implement necessary public policies. Even if the result of government policy is a budget deficit and the accumulation of government debt, there is no problem if full employment is maintained at stable prices, i.e., constant prices or mild inflation. Pensions and other social security benefits should be promoted, as well as the development of airports and the construction of new high-speed railroads, if technically feasible. The technology and production capacity of a country and its people are the constraints for the economy, not its financial resources.

Funding Statement

This research received no external funding.

Acknowledgments

The author would like to thank the reviewers for taking the time and effort necessary to review the manuscript. The author sincerely appreciates all valuable comments and suggestions, which helped the author to improve the quality of the manuscript. Of course, any errors that remain are the responsibility of the author.

Conflict of interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Author contributions

This paper was written by Yasuhito Tanaka alone, with no co-author. He is responsible for setting the research theme, conducting the analysis, and analyzing the data.

References

Blanchard, O. (2022). Deciding when debt becomes unsafe. *International Monetary Fund*. https://www.imf.org/en/Publications/fandd/issues/2022/03/Deciding-when-debt-becomes-unsafe-Blanchard#:%20:text=When

Blanchard, O. (2023). *Fiscal Policy under Low Interest Rates*. The MIT Press.

Domar, E. (1944). The burden of debt and the national income. *American Economic Review*, 34(4), 798–827.

Maebayashi, N., and Tanaka, J. (2020). Limited asset market participation and fiscal sustainability. *The Society for Economic Studies, The University of Kitakyushu Working Paper Series No. 2020-4*. https://www.kitakyu-u.ac.jp/economy/uploads/2020_4.pdf

Otaki, M. (2007). The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 96, 23–29. https://doi.org/10.1016/j.econlet.2006.12.005

Otaki, M. (2009). A welfare economics foundation for the full-employment policy. *Economics Letters*, 102, 1–3. https://doi.org/10.1016/j.econlet.2008.08.003

Otaki, M. (2015). *Keynesian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer. https://doi.org/10.1007/978-4-431-55345-8

Tanaka, Y. (2023a). The budget deficit in an endogenous growth model with bequest and money holdings, *Economic Analysis Letters*, 2, 29-39. https://doi.org/10.58567/eal02010005

Tanaka, Y. (2023b). Money holding and budget deficit in a growing economy with consumers living forever, *Financial Economics Letters*, 2, 30-43. https://doi.org/10.58567/fel02010004

Tanaka, Y. (2024a). An overlapping generations version of Krugman's world's smallest macroeconomic model and fiscal deficit, *Journal of Economic Analysis*, 3, 101-115. https://doi.org/10.58567/jea03040006

Tanaka, Y. (2024b). Budget deficit and money holding when consumers live forever in an endogenous growth model, *Journal of Economic Analysis*, 3, 116-135. https://doi.org/10.58567/jea03040007

Tanaka, Y. (2024c). Budget deficit in a growing economy and impossibility of fiscal collapse: A continuous time analysis, *Central European Economic Journal*, 11, 306-319. https://intapi.sciendo.com/pdf/10.2478/ceej-2024-0020

Yoshino, N., and Miyamoto, H. (2021). Reconsideration of the "Domar condition" to check sustainability of budget deficit. *Public Policy Review, Policy Research Institute, Ministry of Finance, Japan*, 17(3), 1–12.

Weil, P. (1987). Permanent budget deficits and inflation. *Journal of Monetary Economics*, 20, 392–410.

Weil, P. (1988). Overlapping families of infinitely-lived agents. *Journal of Public Economics*, 38, 193–198.