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The Golden Ratio Applied to Financial Gravity Models: Fees, Taxes and Commerce

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ABSTRACT

Recently, some authors have found derivations and applications of the golden ratio in economics science. Based on previous models of Financial Gravity (FG) and generalizing them, this paper proposes the maximization of pure flows after charges as a way for applying the golden ratio in the sales-costs of production ratio in competitive equilibrium, leading to an optimal unitary charge of 0.2361… with many economic applications in monetary economics as well as in finance, public economics and taxation, migration flows, VAT on goods and services and commercial trading. Other applications are also suggested for further research. This paper can be useful for mathematical and applied economists, public financiers, bankers and policy and lawmakers.

KEYWORDS

Financial Services; Monetary Policy; Taxation; VAT; Trading Commerce; Gravity Models

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1. Introduction

Peña (2020a) has stipulated the possible existence of several 'natural constants' in the economics science, something already suggested by other authors, even physicists (Rebentrost et al., 2018) or econophysicists (Săvoiu, G, and Simăn, 2014). The term 'golden ratio' was denominated by Pacioli (1509), father of modern accounting, to an irrational number with plenty of natural applications, which was already discovered several centuries ago by the Ancient Greeks, and takes his phi letter in honor of the sculptor Fidias, who used this proportion in Arts. Recently, Malakhov (2021, 2022) has derived this ratio as the optimal sales-cost of production ratio in competitive equilibrium, inspired by Adam Smith's invisible hand, after assuming that total costs and sales are constant. So, as most of the economic variables depend on the others, once an optimal value is obtained, other values in equilibrium or natural constants can be derived. This is the case in this article. Other applications or derivations of the golden ratio in economics are the finding of phi in inventories (Disney et al., 2004), in economic cycles and sub-cycles (De Groot et al., 2021), games (Schuster, 2017) or in marketing (Thomas and Chrystal, 2013). For a recent review and applications for the financial sector, see Ulbert et al. (2022).

While developed countries are currently experiencing in general a sharp increase in migration in the last years (with a record of 6.5 million new permanent immigrants in OECD countries in 2023, with, respectively, 10% and 28% additional immigrants respect from 2022 and 2019[1], OECD; 2024), the countryside is being depopulated in many countries (EuroStat, 2022), with unsustainable population pyramids. Better migration policies are needed. Additionally, the financial sector has recently entered into a crash, the Global Financial Crisis (GFC) from 2007 to 2009, which has led to one of the worst crises in decades, the Great Recession (GR). This highlights the need of better financial and monetary policies (IMF, 2011). Finally, globalization has emerged in the last decades raising to a global trade openness of 58.51% in 2023 and a recent maximum of almost 63% in 2022 (World Bank, 2024). All these facts underscore the need for better policies on economic flows, with the present paper attempting to address this gap.

This article is divided as follows. After this introduction, a brief literature review of the FG models is presented, and Section 3 presents a generalization of these models applying the golden ratio. Section 4 shows immediate applications in monetary economics, public finance and commerce and additional applications for further research, and Section 5 concludes.

2. Literature Review

The Financial Gravity (FG) models in a strict sense (that is, originally only applied to domestic financial services, without intervening in international issues) were first established by Peña (2021b), based on the exogenous models of Lopez-Laborda and Peña (2018) and Peña (2019), providing the similarities and differences, mainly advantages, from other current financial models of banking's working mechanisms. According to the former paper, commercial banks are seen as providers of services, and so, they can be considered as good-providers as any other non-financial business (Peña, 2019). So, in FG models banks are not mere exchange institutions in contrast to the Intermediary Loanable Fund (ILF) models (Kumhof and Zoltan, 2015). In fact, FG models focus on banks as trading agents who earn a fee by their job, which can lead to non-zero profits. Nonetheless, Internet-based financial services can also be included in these models because here banks do not need to use real resources to provide their services (Peña, 2019 and Jakab and Kumhof, 2019). The value added is given by the provision of a service that satisfies any needs of the consumers (such as safety, liquidity availability, seeking information and services, etc.).

Furthermore, banks in an FG model are neither only mere monetary institutions in contrast to Financing through Money Creation (FMC) models (Kumhof and Zoltan, 2015). In spite of that, there are some similarities with

[1] https://www.migrationdataportal.org/themes/international-migration-flows

the previous models: both models take quantities into account (McLeay, Radia and Thomas, 2014), both theories also consider the potential generation of loans from none deposit and peer-to-peer lending (P2P) is permitted (Lopez-Laborda and Peña, 2018), the FG models also allow banks to have non-zero profits as in Jakab and Kumhof (2019), and finally, both insights consider Central Banks as providers of as many reserves and cash as commercial banks demand at a given reference interest rates (Kumhof and Zoltan, 2015).

In addition to the use of FG models on finance, other models similar to the previous ones have been formulated in papers as Peña (2020b), such as applying preliminary FG models to trade commerce, algorithmic trading and other applications, or also applied to the population flows of migration.

3. The General Model

In this section, a Financial Gravity (FG) model is generalized and some 'charges' are applied to some of the variables, charges that will make sense in the next sections of the application. A general FG model without charges would be:

$$\begin{aligned} \rho R &= R - \varepsilon \\ \rho P &= \varepsilon - P \end{aligned} \tag{1}$$

here ρ is the unitary value added, ρ is the 'pure' flow (without fees nor risk), R is a receipt or entry flow (inflow) and P is a payment or leaving flow (outflow):

$$\rho = \frac{R-P}{R+P}, \varepsilon = \frac{2RP}{R+P} \tag{2}$$

Suppose that a charge is levied with a negative sign for P and with a positive sign for R, then expression (1) would turn into:

$$\begin{aligned} \rho'(1-\alpha)R &= (1-\alpha)R - \varepsilon' \\ \rho'(1+\alpha)P &= \varepsilon' - (1+\alpha)P \end{aligned} \tag{3}$$

where now expression (2) shifts into:

$$\rho' = \frac{(1-\alpha)R-(1+\alpha)P}{(1-\alpha)R+(1+\alpha)P}, \varepsilon' = \frac{2(1-\alpha)R(1+\alpha)P}{(1-\alpha)R+(1+\alpha)P} \tag{4}$$

If the 'pure' flow after charges, ε', but expressed divided by the total net flows, is optimized respect to the charge, it leads to the following program:

$$\underset{\{\alpha\}}{Maximize}\ \delta' = \frac{2(1-\alpha)R(1+\alpha)P}{[(1-\alpha)R+(1+\alpha)P]^2} \tag{5}$$

Before optimizing, some simplifications are performed, concretely, both the numerator and denominator are divided by RP, with y=R/P:

$$\underset{\{\alpha\}}{Maximize}\ \delta' = \frac{2(1-\alpha^2)}{(1-\alpha)^2 y + (1+\alpha)^2\left(\frac{1}{y}\right) + 2(1-\alpha^2)} \tag{6}$$

According to Malakhov (2021, p.11), in competitive equilibrium and after some derivations, the "equilibrium sales-costs of production ratio is equal to the golden ratio", so this is extrapolated to the more general R/P ratio, so y=φ=1.6108..., and then, the only positive root for the program (6) taking into account the previous consideration, is $\alpha^* = -\left(2^{3/2}\sqrt{\sqrt{5}+3} - \sqrt{5} - 5\right) / \left(\sqrt{5}+1\right) \cong 0.2360679775,$ being a maximum of the problem. Alternatively, $\alpha^* = \left(\sqrt{5}-1\right) / \left(3+\sqrt{5}\right)$ or $\alpha^* = \sqrt{5}-2$ take the same value as a positive solution of the program, being the last one the simplest expression among the three of them.

4. Applications

This section shows the main applications found in this paper for FG models to the recent finding of the golden ratio in the competitive equilibrium.

4.1. Finance and Monetary Economics

According to Peña (2021), where a neutral charge on interests is obtained, in this case, where the pure interest is not modified, this author obtains the optimal fee that maximizes the pure interest. Concretely, instead of R they would be the fee-free interest receipts (IR'), and instead of P, the fee-free interest payments (IP'). The pure flow after charges would be the 'implicit' gravity equation for pure interest, and the pure flow before charges would be the optimal pure interest, related to the Central Bank reference rates. In this case, the maximization is made on the unitary pure interests, that is, the latter divided by total interests after fees. The optimization program for the bank would be:

$$\underset{\{\rho_0\}}{Maximize}\ \delta'_B = \frac{2(1-\rho_0)IR'(1+\rho_0)IP'}{[(1-\rho_0)IR' + (1+\rho_0)IP']^2} \tag{7}$$

Following the previous generalization, the interest receipts divided by the interest payments, both before fees, would also be equal to the golden ratio. So, the optimal fee that maximizes the unitary pure interest would also be $\rho_0^* = 0.23606798\ldots$

4.2. Applications for Public Finance

Applying the cash flow method (Poddar and English, 1997) for levying financial services on VAT, the cash inflows (CI) as deposit interests are subsidized, while the cash outflows are taxed, so the tax is levied on the financial margin (cash outflows minus inflows). The equation would be the same as in (7), but with the financial services after taxes in this case:

$$\underset{\{t\}}{Maximize}\ \delta'_{FS} = \frac{2(1-t)IR(1+t)IP}{[(1-t)IR + (1+t)IP]^2} \tag{8}$$

Where $IR = CO = \left(1-\rho_0\right)IR'$ and $IP = CI = \left(1+\rho_0\right)IP'$ and t is the tax rate. Again, the interest receipts divided by the interest payments, both after fees and before taxes, take the value of the golden ratio, so the optimal tax rate that maximizes the unitary implicit rate would be $t^* = 0.23606798\ldots$

4.3. Applications for Taxing Commercial Trade

This subsection considers that the receipts R are the exports and the payments P are imports in an assumed case of a trade coverage rate (ratio of exports and imports) equal to the golden ratio in equilibrium. The variable to maximize is the indicator of trading demand, proposed by Peña (2020b), who found the indicator is related to economic development, being part of a "key for trading development and for evaluating international trade" (p.1). This indicator can also be related to algorithmic trading strategies balanced in profitability and risk (Peña, 2022) and, divided by the GDP, may constitute a better indicator of trade commerce rather than the traditional trade openness indicator (Peña, 2024). In this case, it is expressed in unitary terms, respect to total after-taxes commerce flows. All flows are affected by indirect taxation, a taxation that subsidizes exports and penalizes with a tax the imports:

$$\underset{\{t\}}{Maximize}\ \delta'_{TC} = \frac{2(1-t)X(1+t)M}{[(1-t)X+(1+t)M]^2} \tag{9}$$

With X and M being the exports and imports of the country, respectively. For this example, the optimal tax rate is as in the previous illustration, $t^* = 0.23606798\ldots$.

4.4. Applications for Migration Policies

We can consider the demand of migration in cities i and j by considering the following equation:

$$\varepsilon_J = \frac{2J_{ij}J_{ji}}{J_{ij}+J_{ji}} \tag{10}$$

Where J_{ij} is the migration from i to j (emigration for i) and J_{ji} is the migration from j to i (immigration for i), where $J_{ij} / J_{ji} = \varphi = 1.6108\ldots$ is assumed. The next equation considers that the government of city i aims, as a way to increase the population of city i, to maximize the unitary demand of migration and tries to attract to this city a proportional share α (the charge) additional to the current immigration and aims to reduce the same share of the current emigration. So, the planner's problem would be:

$$\underset{\{\alpha_J\}}{Maximize}\ \delta'_J = \frac{2(1-\alpha_J)J_{ij}(1+\alpha_J)J_{ji}}{\left[(1-\alpha_J)J_{ij}+(1+\alpha_J)J_{ji}\right]^2} \tag{11}$$

so, the optimal share of additional immigrants and of fewer emigrants would be $\alpha_J{}^* = 0.23606798\ldots$.

4.5. Applications for applying VAT to the real economy

In real economics, the optimization of pure flows of manufactures with R and P being the sales (S) and purchases/costs (C), following Malakhov (2021) model. He considered that the sales-cost of production ratio was the golden ratio in equilibrium, $S / C = \varphi = 1.6108\ldots$. In addition, VAT is applied as usual in general goods and services, by applying a proportional tax rate t on sales and allowing the business to credit a proportional tax rate on purchases. So, the maximization of unitary pure output/input flows (OI) of goods and services by the business would be:

$$\underset{\{t\}}{Maximize}\ \delta'_{OI} = \frac{2(1-t)S(1+t)C}{[(1-t)S+(1+t)C]^2} \tag{12}$$

where, again, the optimal tax rate of VAT on general goods and services would be $t^* = 0.23606798\ldots$.

4.6. Additional applications for further research

Other applications may be developed from the findings of Peña (2020b), for instance, alpha being the migrations and R and P the population of a place more and less populated. Additional applications for further research would be the gravity equation of commerce with different GDPs of bigger and smaller countries, as according to equation (5.15) from Feenstra (2002) or applications to transport economics as commuting or to cost-benefit analysis by maximizing the discount rate of the plan. Furthermore, empirical evidence would also be interesting.

5. Conclusions

This paper shows potential applications using Financial Gravity (FG) models to the recent advances of Malakhov (2021, 2022) in the field of competitive equilibrium regarding the appearance of the golden ratio or its inverse in some results of the equilibrium. After proposing a generalized FG model, and including a hypothetical charge that penalizes the receipts and that is added to the payments, a maximization of the pure flows is proposed. In that case, after taking into account the sales-costs of production ratio of equilibrium proposed by the previous author, which is the golden ratio according to him, an optimal charge of $\alpha^* = 0.2360689\ldots$ is obtained. This optimal charge is interpreted, among several possible applications, as the optimal value of the unitary fee of financial services by banks, or the optimal tax rate on trade commerce, general goods or services or in banking. Furthermore, it can be a useful strategy for knowing the optimal share for attracting new population to a city at the same time the same share of emigration is constrained.

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Conflict of interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Author contributions

Conceptualization, formal analysis, funding acquisition, investigation, methodology, project administration, resources, software, supervision, validation, visualization, writing–original draft, and writing–review & editing: Guillermo Peña.

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