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## Optimal Commodity Storage: Privately and Publicly Financed Storage Compared

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### ABSTRACT

Consideration of optimal commodity storage with different discount rates. Finding that, even with a lower discount rate than private storage, optimal government-financed storage may not narrow price fluctuations compared with optimal privately financed storage because a government has to choose a probability of buffer stock failure greater than zero to economize on storage costs that could conceivably become very large. There is no presumption that government financed storage would be larger, with narrower intervention bands and more stable prices, than with privately-financed storage. The welfare enhancing effect of government financed storage compared to private storage is therefore indeterminate.

### KEYWORDS

Optimal commodity storage; Price stabilization; Private storage; Public storage; Storage costs

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## 1. Introduction

At issue is whether the supply of storage is optimally provided through commodity markets or whether publicly financed buffer stock agencies would better approximate optimality. Less diversified than governments, private investors in storage are assumed to apply higher discount rates to expected benefits; but there is no presumption that private storage is less economically efficient. Given finite public funds a probability of buffer stock failure must be set and this can offset the advantage of a lower discount rate. This result is interesting because: i) there is no unique social optimum; ii) there is no presumption that commodity markets inefficiently invest in storage; iii) it plausibly suggests why in the past public buffer stock schemes have failed. That is, will private investors provide socially optimal price-stabilizing commodity storage or will they under-provide for it? To answer this question a model of optimal social storage is required in which to compare the amount of storage that would be provided by an idealized social planner with amounts supplied through private futures markets.

Following Wright and Williams (1982), Gouel (2013), and Scheinkman and Schechtman (1983), the model developed here is partial equilibrium with disturbances to market equilibrium emanating from only one side of the market, the demand-side, rather than in these other papers the supply-side. The main findings are not sensitive to which side of the market price disturbances emanate, or, as in Edwards and Hallwood (1980), both sides simultaneously. While supply-side disturbances are typical in agricultural markets, the assumption of demand-side disturbances is more applicable to metals and other mined products where supply is not much affected by vagaries of the weather. As Scheinkman and Schechtman (1983) assume rational expectations and risk neutrality on the part of private sector buyers and sellers, private storage is expected to be socially optimal, leaving no argument in favor of publicly financed storage. Gouel (2013) likewise assumes rational expectations but argues that socially financed storage may increase consumer welfare if consumers – say, of food – are risk averse.

The innovation of this paper is to consider the case for (and against) publicly financed storage based upon a different market imperfection: differences in discount rates between private and public sector investors in storage. Owing to lesser diversification across investment projects private discount rates can be assumed to be higher than social discount rates. Also relevant is the observation of Townsend (1977), namely, if market prices of a given commodity follow a random walk, any publicly financed buffer stock with fixed upper and lower intervention prices is bound to fail.<sup>1</sup> For this reason any publicly financed buffer stock must choose a ‘probability of failure’ greater than zero, otherwise, potentially, it will need unlimited funding. Building on the work of Oi (1961) and Waugh (1966), Massell (1969) attempted to answer the question of the optimum intervention price limits but did so on the restrictive assumption that the main objective was that a buffer stock agent simply aimed for gross revenue to cover gross operating costs. Hueth and Schmitz (1972) introduced the idea of optimizing buffer stock intervention prices with reference to maximizing the sum of consumer and producer surpluses but left commodity stockpiling costs out of consideration – an integral variable considered here.

It is assumed that a market includes all the producers and consumers of a given commodity, and that the welfare objective is to maximize “world welfare” as measured by the sum of producer and consumer surpluses less price-stabilization costs. Net benefit is maximized when the marginal cost of storage is equal to the discounted expected marginal benefit yielded by a reduction in price instability. One of the main tasks in this paper is to establish the relationships between intervention price limits (as determined by private investors, or, in the case of a publicly financed buffer stock, a public agent) and the marginal cost and expected marginal benefit of storage.

The paper continues in section 2 with a discussion of privately financed storage. Section 3 introduces the relationship between intervention price limits and the residuals between demand and supply. Section 4 discusses the concept of the probability of buffer stock failure that is crucial in any comparison of the social optimality of privately and publicly financed commodity storage. This is followed with an account of the welfare effects of price stabilization in section 5. Sections 6, 7 and 8 discuss the related topics of the optimal range of intervention prices, the cost of price stabilization and optimization of commodity storage. Section 9 compares optimal private and social storage. Section 10 concludes that the amount of storage induced through futures markets may be less or more than that aimed for by an idealized social planner, and that there is no ‘universal’ concept of price-stabilizing optimal social storage against which to compare the amount and effects on market prices of storage induced through private markets.

## 2. Private storage

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<sup>1</sup> The possibility of buffer stock failure is not recognized in Newbery and Stiglitz (1982).

A model of optimum storage by private investors is developed by Pindyck (2001). Storage involves a 'convenience yield' but is costly. Crucially, investors in storage, all of whom being driven by the profit motive (and engaged in by both third parties operating in futures markets as well as suppliers and buyers of the physical commodity), demand a risk premium because future spot prices are unknown and could deviate from expectations. Pindyck (2001, page 18) points out that the risk adjusted discount rate,  $\rho_T$ , should exceed the risk-free interest rate,  $r_T$ . This in turn implies that usually the futures price should not be as high as the *expected* spot price because the difference between the two yields an expected profit for investors in commodity storage. Moreover, in an efficient market, and averaged over time, the expected spot price at time  $t$  for period  $t + n$  is the best unbiased predictor of the spot price that occurs in period  $t + n$ . Researchers find that futures prices outperform a random walk as predictors of spot prices, and that they also outperform any selected linear models (Reeve and Vigfusson, Reichsfeld and Roache, 2011).

The need to reward investors in commodity stocks with a profit explains why the futures price is typically above the current spot price. According to Reeve and Vigfusson (2011) most open interest in commodity futures is held by speculators rather than hedgers – the latter either having (suppliers) or expecting to have (buyers) a long position in the physical commodity. Reeve and Vigfusson (2011) report that the fraction of open interest held by speculators in 8 major commodity markets can vary a lot, but for 6 of the 8 commodities they report on (copper, cotton, corn, sugar, wheat and soybeans) 60-percent is a good representative figure, the proportion was much higher for natural gas and WTI crude oil.

Lee and Blandford (1980) apply a linear econometric model to the world cocoa and copper markets finding that with price stabilization, producers' total revenue would increase but that the buffer stock needed to stabilize price on the average price would have been 'very expensive'. Hughes-Hallett (1986) and Ghosh, Gilbert and Hughes-Hallett (1984) also found that buffer stock operations in the markets they studied would have benefited producers but would also have been expensive. Gilbert (1987 and 1996) says that the few international commodity agreements that have operated broke down for various reasons: including high costs and lack of agreement between the members on critical issues. However, neither he nor anybody else have discussed the idea that private storage may already be close to socially optimal - rendering the addition of public buffer stocks a sub-optimal activity. Interestingly, Zant (1997) investigates private stockholding in the Indian natural rubber market finding it to be 'price stabilizing'; and that in his simulations the introduction of a price stabilizing public buffer stock would not necessarily have been more effective. Indeed, a buffer stock would have reduced the price elasticity of private stockholding thereby reducing the amount of private storage.

The question is then "is private price-stabilizing commodity storage near being socially optimal or would storage by a theoretical social planner outperform it?" This is akin to asking whether private markets work well or whether some sort of corrective intervention is needed to offset a market failure. Put differently, theoretically, does private speculation in stockholding work well enough to invalidate the need for social storage?

To investigate this question a model of the optimum buffer stock is required which is the task now turned to. The model is presented largely in graphical form.

### 3. Intervention prices and residuals between demand and supply

Price stabilization deals in the residuals between supply and demand. Thus,

$$R_t = D_t - S_t \quad (1)$$

where, at some given price,  $R_t$  is the residual,  $D_t$  market demand, and  $S_t$  market supply from new production. Unless a commodity buffer stock supplies the residual when demand is greater than supply or stores the residual when demand is less than supply, market price will change.

It is assumed that in the long-run neither positive nor negative stock accumulation occurs. Accordingly, expected demand equals expected supply. However, residuals can occur in the current time period if  $D_t$  and  $S_t$  are subject to unforeseen variations and that their covariance is less than unity. With normally distributed  $D_t$  and  $S_t$ ,  $Var(R_t)$  is:

$$Var(R_t) = Var(D_t) + Var(S_t) - 2Cov(D_t, S_t) \quad (2)$$

The magnitude of  $Var(R_t)$  is determined by the factors that determine the magnitudes of  $Var(D_t)$  and  $Var(S_t)$  as well as their covariance. These factors include the degree of fluctuation in aggregate demand or industrial production or some such index, the elasticity of demand with respect to these indices, as well as

fluctuations in climate and the incidence of animal and plant diseases in the case of agricultural goods, and the incidence of mining disasters, strikes and transportation holdups in the case of minerals.

Low positive covariance of demand and supply is a factor that contributes to a high  $Var(R_t)$ . Low covariance causes market price to fluctuate more than if it were higher. In a market system, positive covariance is attained through final consumption and production adjusting to changes in market price. For instance, a bumper harvest causes prices to fall and demand to rise so changes in  $D_t$  and  $S_t$  are positively associated. Covariance of  $D_t$  and  $S_t$  is low where the market signaling system is impaired, for example, because of lags in the adjustment of demand or supply to current market price, or, because they are functionally related to some other variables, such as expected prices that are themselves not perfectly correlated with current price because the elasticity of price expectations differs from unity.

#### 4. Probability of buffer stock failure

How the residuals between demand and supply are dealt with differs between a social planner running a publicly financed buffer stock agency and private speculators in commodity stocks operating in futures markets. The former is assumed to target maximizing net social welfare of commodity storage – as discussed in the next section, while private investment in commodity storage is for profit.

The probability of failure to support the lower intervention price is contained in the left tail of the probability distribution of  $R_t$ , and the probability of failure to impose the upper intervention price on the market in times of excess demand is contained in the right tail. The probability of buffer stock failure,  $P_F$ , is the sum of the tail probabilities.

Choice of a low  $P_F$  means that the tail probabilities are relatively small meaning that the intervention price limits are relatively wide apart. The choice of a probability of buffer stock failure is likely to depend on: a) the preferences of politicians and/or their agents running a buffer stock not to be seen as ‘failures’ in the sense that a buffer stock scheme breaks down - in which case they would choose a low probability of buffer stock failure with the associated wide intervention price bands; and b) constraints on funding – the more limited are funds the less is the amount of intervention that can take place and so the wider must be the intervention price limits and the lower is the probability of buffer stock failure.

Once a certain  $P_F$  has been chosen, and for a given  $Var(R_t)$ , the maximum physical quantity that a buffer stock can take off the market,  $m_t$ , is determined. The required funding of a buffer stock agency also depends on the level of intervention prices because these determine the cost of acquisition.

#### 5. Welfare effects of price stabilization

Analysis of a two sector stochastic commodity market model shows that price stabilization raises ‘world welfare’.<sup>2</sup> Assume that

$$D_1 = k_1 + bP_t + u_t \quad (3)$$

$$D_2 = k_2 + bP_t + u_t \quad (4)$$

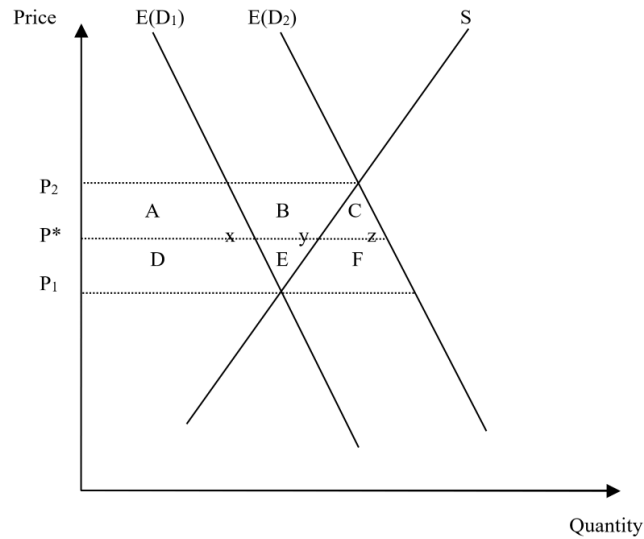
$$S_t = n + cP_t \quad (5)$$

where  $P_t$  is current price;  $u_t$  a normally distributed random variable with expected value = 0;  $S_t$  is a deterministic supply curve;  $k_1$ ,  $k_2$ ,  $n$  and  $c$  are positive constants and  $b$  is a negative constant.  $k_1$  and  $k_2$  are expected to occur with equal probability. Figure 1 shows a set of supply and expected demand curves following from these assumptions.

The expected welfare loss caused by market price moving systematically between  $P_1$  and  $P_2$  instead of remaining fixed at  $P^*$  is equal to the areas of the triangles  $C + E$ . Examination of Figure 1 shows

$$\Delta CS = (A + B + C - D) \quad (6)$$

<sup>2</sup> However, price stabilization is not Pareto optimal since it involves a reduction in welfare for one side of the market – the side that causes the price instability, in the sense of being the origin of the shifts in the supply or demand curves, will gain from price stabilization even after making compensating payments to the trading partners.



**Figure 1.** Expected Changes in market prices: variable demand.

where  $\Delta CS$  is the expected loss in consumers' surplus caused by destabilized prices relative to keeping price fixed at  $P^*$  and

$$\Delta PS = [D + E - (A + B)] \tag{7}$$

where  $\Delta PS$  is the expected loss in producers' surplus. The loss in world welfare,  $WW$ , caused by destabilized prices is

$$WW = \Delta CS + \Delta PS \tag{8}$$

therefore,

$$\Delta WW = C + E \tag{9}$$

which is positive so that the side of the market gaining from price stabilization, in this case consumers, could compensate the losers and still enjoy a higher level of welfare.

Repetition of this analysis for stochastic supply and deterministic demand would show that destabilized prices also reduce world welfare and that price stabilization, while increasing world welfare, is not Pareto optimal because consumers' surplus is reduced.

A feasible probability distribution of  $P_t$  is shown in Figure 2.a. The distribution is bimodal because of the shift factors  $k_1$  and  $k_2$ .  $P^*$  is the single market clearing price that would have to be set by a buffer stock to stabilize price without secularly accumulating or de-cumulating stocks because  $xy = yz$  in Figure 1. Also,  $P^*$  is the price where world welfare loss is zero and if  $P_t \neq P^*$  a world welfare loss occurs.

The world welfare loss function is shown in Figure 2.b. The function has positive slope for values of  $P_t > P^*$  and a negative slope for values of  $P_t < P^*$  because as market price moves away from  $P^*$  the welfare loss triangles in Figure 1 become larger and do so at an increasing rate so that the second derivative of the welfare loss function is positive. It can be shown that the rate of increase in welfare loss is greater the more inelastic are the supply and demand curves.<sup>3</sup> The welfare loss function is constrained because the maximum welfare loss occurs when  $P_t$  is so low that supply is zero or so high that demand is zero.

The expected world welfare loss,  $E(WWL)$ , caused by unstable prices is given by

$$E(WWL) = \int_{p_1}^{p_2} P(P_t) \cdot WL(P_t) \cdot dP_t \tag{10}$$

<sup>3</sup> The proof of these propositions is as follows:  $A = \frac{1}{2}bh$  where  $A$  is the area of a triangle. But  $h = ab$  where  $a$  is a positive constant. Therefore,  $A = \frac{1}{2}ab^2$ , so  $dA/db = ab$  and  $d^2A/db^2 = a > 0$ . Furthermore, for any given lateral shift in the demand or supply curves,  $b$  is larger the more inelastic are the demand and supply curves.

Figure 2.a. Probability Distribution of Market Price.

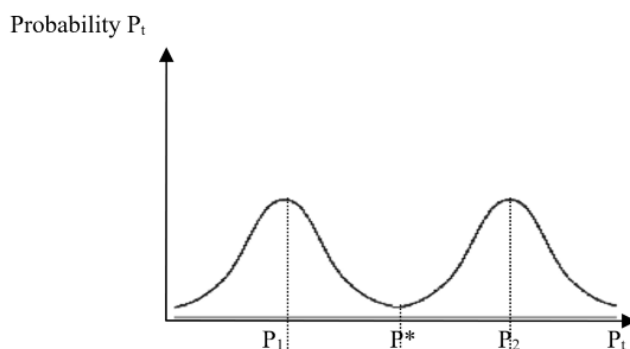
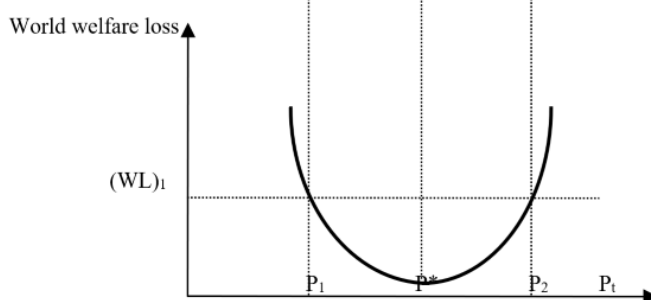


Figure 2.b. World welfare loss function.

**Figure 2.** Probability Distribution of Market Price and World welfare loss function.

where  $WL$  is the welfare loss incurred by unstable prices and  $P(P_t)$  the probability density function of price. Probability  $P_t$  is given by the area between the extremes of market price and under the probability distribution of  $P_t$  in Figure 2.a., for instance, the area above  $P_1 \leq P_t \leq P_2$ . The probability of  $P^*$  occurring becomes smaller as probability  $P_t$  increases so that the probability of a welfare loss arising becomes larger.  $E(WWL)$  is given by the height of the welfare loss function above the horizontal axis in Figure 2.b.: for instance,  $(WL)_1$  when  $P_t = P_1$  or  $P_2$ . Both probability  $P_t$  and  $WL$  are increasing functions of price variability measured by the difference between the extremes of market prices. Thus, the mean value of  $E(WWL)$  will become larger as the upper and lower limits of market price move apart.

The functional relationship between  $E(WWL)$  and increasing ranges of expected market prices is shown in Figure 3.  $p^u$  is the upper limit of expected market price and  $p^l$  its lower limit.  $OM$  is of positive slope because both probability  $P_t$  and  $WL$  are increasing functions of instability measured by  $p^u - p^l$ . Although the second derivative of  $WL$  is always positive, the second derivative of  $OM$  varies between positive and negative at the turning points of the probability distribution of  $P_t$ — so the second derivative of  $OM$  is not necessarily positive.  $OM$  is not drawn as a linear function to reflect the possibility of changes in the sign of the second derivative.

## 6. Buffer stock intervention prices

The immediate objective of a buffer stock is to stabilize price within the market determined limits. The difference between its selling price ( $P_s$ ) and buying price ( $P_b$ ) is:

$$\pi = P_s - P_b \quad (11)$$

An expected world welfare benefit,  $E(B)$ , arises if the buffer stock sets  $\pi$  between the limits of expected market prices and the size of this expected benefit for  $\pi = \pi_1$  in Figure 3 is  $E(WWL)_1 - E(WWL)_2$ .

The expected benefit function for  $P_1 \leq P_t \leq P_2$  is drawn in Figure 4.  $KK_1$  has negative slope because  $E(WWL)$  declines as  $\pi$  becomes smaller; that is,  $E(B)$  increases. The concavity (downward) of the expected benefit function will be more marked the greater is the price inelasticity of supply and demand (because the second derivative of the  $WL$  function with respect to  $p^u - p^l$  will be larger and will tend to dominate the effect of changes

in the second derivative of probability  $P_t$ ).  $KK_1$  touches the horizontal axis at  $K_1$ , when  $\pi = P_2 - P_1$  because a buffer stock is not intervening in the market.  $E(B)$  is maximal at point  $K$ , because no price instability or welfare loss occurs (provided that the intervention price is set at  $P^*$ ).<sup>4</sup>

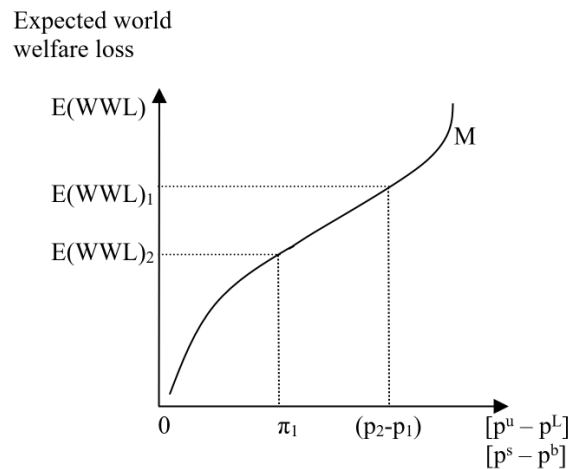


Figure 3. Expected welfare loss and the range of market prices.

## 7. The relationship between $\pi$ and the gross costs of price stabilization

Gross stabilization cost,  $G$ , incurred by a buffer stock is defined as

$$G = H + tUi \quad (12)$$

where  $H$  is the gross warehousing cost of storage (for simplicity assumed to be a constant fixed cost whether or not commodities are stockpiled);  $t$  is time measured in years for which the commodity is held in the stockpile;  $U$  is the total value of funds tied-up in the stored commodity (this will depend on purchase price and volume stored); and  $i$ , the annual rate of interest (the opportunity cost of funds). There is a linear negative relationship between gross stabilization cost incurred by a buffer stock and the size of  $\pi$ . The negative relationship follows because the greater is the degree of price instability allowed by a buffer stock, the lower is the price and volume of the commodity taken off the market and stored. The linearity of the function follows from the assumption that the supply and demand curves are linear – this point is proved in Part A of the Appendix.

Figure 4 also shows a negative linear relationship between  $G$  and  $\pi$  drawn for a given probability of buffer stock failure. Point  $T_1$  is fixed where the range of intervention prices equals the range of expected non-regulated market prices, so that no storage cost is incurred.  $TT_1$  touches the vertical axis at  $T$  showing that maximum gross stabilization costs are incurred when  $\pi = 0$ .<sup>5</sup>

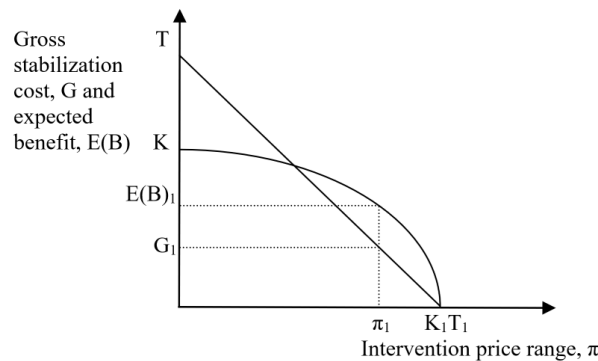
## 8. The optimal $E(B)$ , $G$ and $\pi$

It has been established that  $dE(B)/d\pi$  and  $dG/d\pi$  are both negative and that the positions of these functional relationships are determinate. The two functions are combined in Figure 4 to determine the optimal intervention price range and hence the optimal degree of price instability and gross costs of price stabilization for

<sup>4</sup> The position of  $KK_1$  depends upon the degree of price instability in the unregulated market price. If price instability, as measured by  $p^u - p^l$ , increases,  $KK_1$  will move away from the origin. This implies that a given value of  $\pi$  is associated with a larger expected benefit the more unstable is the unregulated market price. Geometrically, the further to the right is  $P_2 - P_1$  in Figure 3, the greater is the reduction in  $E(WWL)$  for any given value of  $\pi$ .

<sup>5</sup> If either or both of the supply and demand curves of Figure 1 are of greater elasticity than actually drawn, the costs of stabilization will increase less rapidly as  $\pi$  is reduced so that  $TT_1$  will pivot anti-clockwise on  $T_1$  - see part B of the Appendix. An increase in the range of market price shifts  $TT_1$  parallel and to the right. Finally, if  $P_F$  is set at a lower value,  $TT_1$  will pivot clockwise on the point  $T_1$ , higher storage costs being incurred to give better insurance that some given intervention price will set will be secure.

some chosen probability of buffer stock failure, nominal prices, interest rate, periodicity of supply and demand fluctuations, and elasticity of supply and demand.



**Figure 4.** Stabilization costs,  $G$ , and intervention price range,  $\pi$ .

From the particular set of relationships drawn in Figure 4 a buffer stock should be established because  $E(B) > G$  for some values of  $\pi$ . The optimum  $\pi$  and  $G$  are determined at the point where net expected benefit is maximized and this occurs at the point where expected marginal benefit equals marginal cost. In Figure 4,  $E(MB) = MC$  when  $\pi = \pi_1$ ,  $E(B) = E(B)_1$ , and  $G = G_1$  so that the maximum net expected benefit is maximized at  $E(B)_1 - G_1$ .

## 9. Comparing optimal social and private storage

An increase in the required rate of return on commodity storage – the term ‘ $i$ ’ in equation (12) will pivot  $TT_1$  clockwise in Figure 4. The result is that the expected net benefit of storage is reduced as the width of the intervention price limits,  $\pi$ , widens. This is a critical point because the required private rate of return on investment in storage is likely to be higher than the required rate of return to a publicly financed buffer stock agent because the suppliers of public funds to the agency are likely to be more diversified than are private investors – governments tend to hold broader investment portfolios than do private investors. The implication then is that considering relative costs of funds private investors will under-invest in commodity stocks compared to the social optimum.

However, this is not the end of the matter. If the parties to a publicly financed buffer stock scheme happened to decide upon a lower value for the probability of buffer stock failure (wider intervention price bands) for given funding than associated with  $TT_1$  in Figure 4,  $TT_1$  will again pivot clockwise on  $T_1$  with the same results as discussed in the previous paragraph.

There is then no unique standard of socially optimal storage against which to compare optimal private storage. Thus, private storage is optimized using a higher rate of discount to take account of risk, but it would be inappropriate to compare the resulting (smaller) amount of storage with the higher amount of publicly financed storage encouraged by the latter’s lower required rate of return. This is because social investment in commodity stocks requires the setting of a probability of buffer stock failure. The choice of a low  $P_F$  implies relatively wide intervention price limits,  $\pi$ , and from Figure 4 lower expected net benefits.

## 10. Conclusions

This paper has discussed the optimality of investment in commodity storage by private investors comparing this optimum with optimum social storage. It was necessary to consider the factors that determine the optimal degree of price variability between intervention price limits in price-unstable commodity markets. The amount of privately financed private storage is shown critically to depend on the required rate of return for investment in it. This required rate of return is very likely to be higher than that required by governments investing in commodity buffer stocks because the former are less well diversified.

However, relative required rates of return are only a part of the calculation. This is because it is necessary for the parties to a proposed publicly financed buffer stock scheme to set their acceptable probability of buffer stock failure. Unless this is done, and on the assumption of normally distributed market residuals, an unlimited supply of funds would be required to make the buffer stock effective in all eventualities. Once a probability of buffer stock failure is set, the degree of price instability, gross stabilization costs and net expected benefit are optimized when



the marginal cost of storage is equal to the expected marginal social benefit resulting from price stabilization. However, there can be no presumption that privately financed commodity storage must be less than this socially determined optimum especially if a low probability of buffer stock failure is set.

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## Declaration of Competing Interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

## Appendix

### Part A

The negative linear relationship between  $G$  and  $\pi$  is shown as follows. Let

$$G = a + bQ^* \quad b > 0 \quad (A1)$$

where

$$Q^* = Q^s - Q^d \quad \text{at } p_b \quad (A2)$$

$$Q^s = c + dp_b \quad d > 0 \quad (A3)$$

$$Q^d = e - fp_b \quad f > 0 \quad (A4)$$

Substituting (A3) and (A4) into (A2) and rearranging

$$Q^* = (c - e) + (d + f)p_b \quad (A5)$$

Therefore,

$$G = a + b(c - e) + b(d + f)p_b \quad (A6)$$

By definition

$$p_b = p^* - \frac{1}{2}\pi$$

Thus,

$$G = a + b(c - e) + b(d + f)p^* - \frac{1}{2}b(d + f)\pi \quad (A7)$$

Therefore,

$$\frac{dG}{d\pi} = -\frac{1}{2}b(d + f) \quad (A8)$$

which defines the linear negative relationship between  $G$  and  $\pi$ .

### Part B

Stabilization cost and demand and supply elasticity.

The argument that stabilization costs fall at high demand and/or supply elasticities is easily shown. Differentiating equation (A8) of the appendix gives

$$\frac{\delta^2 G}{\delta \pi \delta d} = -\frac{1}{2}b < 0 \quad (A9)$$

implying that the more elastic is the supply function the lower are the costs of storage for a given value of  $\pi$ .  
And

$$\frac{\delta^2 G}{\delta \pi \delta f} = -\frac{1}{2}b < 0 \quad (A10)$$

implying that the more elastic is the demand function the lower are the costs of storage for a given value of  $\pi$ .

## References

- Edwards, R. and Hallwood, C. P. (1981), 'The Determination of Optimum Buffer Stock Intervention rules', *Quarterly Journal of Economics*, February, 151-166. <https://doi.org/10.2307/1884609>
- Ghosh, S., Gilbert, C.L. and Hughes Hallett, A.J. (1984). Commodity Market Stabilization: A Comparison of Simple and Optimal Intervention Strategies in the World Copper market, *Journal of Policy Modeling*, 6 (4), November, 555-572. [https://doi.org/10.1016/0161-8938\(84\)90006-1](https://doi.org/10.1016/0161-8938(84)90006-1)
- Gilbert, C. L. (1987), International Commodity Agreements: Design and Performance, *World Development*, 15 (5), May, 591-616. [https://doi.org/10.1016/0305-750X\(87\)90005-2](https://doi.org/10.1016/0305-750X(87)90005-2)
- Gilbert, C.L. (1996), International Commodity Agreements: An obituary notice *World Development*, 24 (1), January 1996, 1-19. [https://doi.org/10.1016/0305-750X\(95\)00121-R](https://doi.org/10.1016/0305-750X(95)00121-R)
- Gouel, C. (2013), Rules versus Discretion in Food Storage Policies, *American Journal of Agricultural Economics*, 95 (4), 1029-1044. <https://doi.org/10.1093/ajae/aat016>
- Hueth, D., and Schmitz, A. (1972) International Trade in Intermediate and Final Goods: Some Welfare Implications of Destabilized Prices, *Quarterly Journal of Economics*, LXXXVI, 351-65. <https://doi.org/10.2307/1880797>
- Hughes Hallett, A.J. (1986), Commodity Market Stabilisation and 'North-South' Income Transfers: An empirical investigation, *Journal of Development Economics*, 24 (2), December, 293-316. [https://doi.org/10.1016/0304-3878\(86\)90094-5](https://doi.org/10.1016/0304-3878(86)90094-5)
- Lee, S. and Blandford, D. (1980), An analysis of International Buffer Stocks for cocoa and copper through dynamic optimization, *Journal of Policy Modeling*, 2 (3), September, 371-388. [https://doi.org/10.1016/0161-8938\(80\)90029-0](https://doi.org/10.1016/0161-8938(80)90029-0)
- Massell, Benton F., (1969), Price Stabilization and Welfare, *Quarterly Journal of Economics*, LXXXIII, May, 285-97. <https://doi.org/10.2307/1883084>
- Newbery, D.M.G. and Stiglitz, J.E. (1982), Optimal Commodity Stock-Piling Rules, *Oxford Economic Papers*, New Series, 34 (3), 403-427. <https://doi.org/10.1093/oxfordjournals.oep.a041559>
- Oi, Walter Y. (1961), The Desirability of Price Instability under Perfect Competition, *Econometrica*, XXIX, 58-64. <https://doi.org/10.2307/1907687>
- Pindyck, R. S. (2001), The Dynamics of Commodity Spot and Futures Markets: A Primer, *The Energy Journal*, 22 (3), 1-29. 10.5547/ISSN0195-6574-EJ-Vol22-No3-1
- Reeve, T.A. and Vigfusson, R.J. (2011), Evaluating the Forecasting Performance of Commodity Futures Prices, Board of Governors of the Federal Reserve System, *International Finance Discussion Papers*, Number 1025. <https://dx.doi.org/10.2139/ssrn.1912969>
- Reichsfeld, D.A. and Roache, S.K. (2011), Do Commodity Futures Help Forecast Spot Prices? IMF Working Paper WP/11/254. <https://ssrn.com/abstract=1956401>
- Scheinkman, J.A. and Schechtman, J. (1983), A Simple Competitive Model with Production and Storage, *Review of Economic Studies*, 50 (3), 427-441. <https://doi.org/10.2307/2297674>
- Townsend, R. (1977), The Eventual Failure of Price Fixing Schemes, *The Review of Economic Studies*, 14 (3), 190-199. [https://doi.org/10.1016/0022-0531\(77\)90092-8](https://doi.org/10.1016/0022-0531(77)90092-8)
- Waugh, F. V. (1966), Consumer Aspects of Price Instability, *Econometrica*, XXXIV, April, 504-08. <https://doi.org/10.2307/1909949>
- Wright, B.D. and Williams, J.C. (1982), The Economic Role of Commodity Storage, *The Economic Journal*, 92 (367), September, 596-614. <https://doi.org/10.2307/2232552>
- Zant, W. (1997), Stabilizing Prices in Commodity Markets: Price Bounds versus Private Stockholding, *Journal of Policy Modeling*, 19 (3), June, 253-277. [https://doi.org/10.1016/S0161-8938\(96\)00050-6](https://doi.org/10.1016/S0161-8938(96)00050-6)