Journal of Economic Analysis 2023 2(4) 140-153

Journal of Economic Analysis

Homepage: https://www.anserpress.org/journal/jea

Critical condition for deposit insurance to partially or fully substitute for raising capital under cyclical economic environment

Hong Mao [a], Krzysztof Ostaszewski [b, *], Jin Wang [c]

[a] *Shanghai Second Polytechnic University, Shanghai, China*
[b] *Department of Mathematics, Illinois State University, Illinois, USA*
[c] *Office of Information Security, Beijing, China*

ABSTRACT

Regulation of financial institutions has two key purposes: Solvency (prudential regulation) and consumer protection. Prudential regulation is implemented mainly by capital requirements, but governments also provide insurance for customer deposits, as a backup tool. In this article, we discuss the critical conditions for deposit insurance and capitalization to act as substitutes for each other, under cyclical economic environment. We make two assumptions. The first one is that deposit insurance is fairly priced and there is no moral hazard. The second one is that insurance creates incentives for moral hazard among insured banks, resulting in increased risk taking. We also discuss the critical conditions for deposit insurance and capitalization to be complementary under different proportions of deposit insurance.

KEYWORDS

Deposit insurance; capitalization; critical conditions; sensitivity analysis

* Corresponding author: Krzysztof Ostaszewski
E-mail address: krzysio@ilstu.edu

ISSN 2811-0943
doi: 10.58567/jea02040008

Received 29 May 2023, Accepted 28 June 2023, Available online 10 July 2023, Version of Record 15 December 2023

1. Introduction

Regulation of financial institutions has two key purposes: solvency (prudential regulation), and consumer protection. Prudential regulation is implemented mainly by capital requirements, but governments also provide insurance for customer deposits, as a backup tool. Modern approaches to capital requirements are all based on the risk undertaken by the financial intermediaries, in both banking and insurance. But governments also provide deposit insurance (or guaranty funds in insurance), which serve as tools of consumer protection, but also as tools of prudential regulation by preventing so called bank runs, i.e., situations when customer withdraw their balances in large amounts from a financial intermediary due to fear of bankruptcy of that financial intermediary. Our work is concerned with interaction between capital requirements and deposit insurance for banks (or guaranty funds in the case of insurance companies).

There is a significant body of literature, including theoretical and empirical studies, on deposit insurance. Mao and Cheng (2020) give literature review. Starting with Merton (1977), a vast literature has used the arbitrage pricing method to determine the fair insurance premium. The arbitrage pricing method assumes. among other things, that financial markets are complete, that provider of deposit insurance has perfect information about the risk of banks' assets, and it can value accurately banks' assets, and moral hazard is explicitly rules out. Diamond and Dybvig (1983) propose that deposit insurance is an effective method to prevent panic run on a bank. Matutes and Vives (1996) believe that deposit insurance can prevent systemic crises, expand the deposit base, and improve social welfare. Bhattacharra, Boot and Thakor (1998) think that deposit insurance can not only prevent a basic run on a bank, but also can prevent a speculative run. Fama (1985) and James (1987) think that it is difficult to fairly price the deposit insurance because of information asymmetry. Kreps and Wacht (1971) point out that it is necessary to consider risk-based premium and co-insurance in order to avoid moral hazard. Kantas (1986) discuss how to jointly carry out the risk-sensitive pricing of deposit insurance and the discount window in an environment where banks have private information. Santos (2001) reviews the theoretical literature on bank capital regulation and the literature on the design of the financial system and the existence of banks. He analyzes the market failures that justify banking regulation and also analyzes the mechanisms that have been suggested to deal with these failures. Santos (2006) reviews the literature on the two arrangements that most countries have adopted to insure banks against liquidity shocks, a lender of last resort and deposit insurance, and compares the design of these arrangements across countries. Fungacova, Weill and Zhou (2010) examine how the introduction of deposit insurance influences the relationship between bank capital and liquidity creation. Their findings suggest that the deposit insurance scheme exerts a limited impact on the relationship between bank capital and liquidity creation and does not change the negative sign of the relationship. Demirgüc–Kunt et al. (2015) provide a comprehensive, global database of deposit insurance arrangements as of 2013. They extend their earlier dataset by including recent adopters of deposit insurance and information on the use of government guarantees on banks’ assets and liabilities, including during the recent global financial crisis. They also produce a Safety Net Index capturing the generosity of the deposit insurance scheme and government guarantees on banks’ balance sheets. Diamond et al. (2017) begin with a framework for organizing the theories of intermediation. Then they draw out the implications for what the theories say about regulation and note that in many respects the motivation for regulation has been only loosely tied to the theory of intermediation. Finally, they posit some open questions for regulators and economists interested in banking. Liu, et al. (2018) present a closed form deposit insurance pricing formula under GARCH framework and a method for estimating the deposit insurance pricing and evaluating deposit insurance premium rate from market data. Their results indicate that the deposit insurance premium rate under GARCH is lower than its BS counterpart during high-risk periods. Most of above literature discuss how to price deposit insurance fairly and how to prevent the risk taking and moral hazard from perspective of deposit insurers or regulators. Since the stockholders have incentive to monitor banks to ensure perfect solvency, it is necessary for banks to select optimal scheme to avoid or decrease risks at minimum costs from risk management perspective. In this article, we discuss deposit insurance and capitalization mainly from perspective of insured banks. We also discuss the role of partial insurance and capital regulation to decrease moral hazard.

The main risk faced by banks is credit risk: a borrower will default on some type of debt by failing to make payments obligated to. The general method to transfer such kind of risk is to buy deposit insurance. However, buying deposit insurance requires payin deposit insurance premium and it is likely to cause moral hazard especially when the insurance premium is not determined fairly. Another way for banks to avoid credit risk is to raise capital. However, raising capital requires paying capital cost and generally, capital is costly. Banks need to evaluate which of thesetwo approaches is more economical. Bond and Crocker (1993) examine the complementary relationship between bank capitalization and deposit insurance as tools that reduce the exposure of risk-averse depositors to a bank's random portfolio returns. They also point out that deposit insurance and capitalization are not perfect substitutes because of presence of monitoring costs. However, they only discuss how to optimize deposit insurance and determine the categorizing premium of deposit insurance under different capital levels. We think that it is necessary to jointly determine optimal deposit insurance and capitalization, since the cost of deposit insurance, capital level, and capital cost are all related to each other. Mao and Cheng (2020) discuss how to optimize capitalization and deposit insurance strategies under cyclically economic environment and under consideration of moral hazard inherent in deposit insurance. The objective of banks is to minimize the sum of capital cost, expected loss of banks' bankruptcy and the opportunity loss of partial deposit insurance. In this article, we present that capitalization and deposit insurance can be substituted with each other under some conditions. We discuss these conditions and determine the critical bound of raising capital to instead of deposit insurance under different ratio of capital cost.

2. Deciding whether to buy deposit insurance or raise capital in order to manage credit risk

2.1. Models

2.1.1. Stochastic models of assets and liabilities of banks

In this section we will establish models to decide whether to buy deposit insurance or raise capital in order to manage credit risk. The models presented are consistent with current literature of modeling financial markets and financial institutions with stochastic processes describing the key variables. Such models have certain limitations. For example, models based on standard Brownian motion effectively assume that continuously compounded rates of return follow a normal distribution, but empirical data indicates that the normal distribution underestimates probabilities of occurrence of extreme tail events. Also, Vasicek model used as a part of our approach has been criticized as a tool for modeling interest rates because under Vasicek's model, it is theoretically possible for the interest rate to become negative. Interestingly, both the 2008 Credit Crisis, and the 2020 Covid-19 induced crisis have brought about negative interest rates on certain risk-free securities, and thus vindicated the Vasicek model to a degree.

We assume that deposit insurance and capitalization are substituted if not considering moral hazard. We have known that the main factor affecting deposit insurance premium is the credit risk faced by banks. Here, we use dynamic default probability $Q(t)$ to express the credit risk at time t, $t = 1, 2, \dots T$ faced by banks. We assume that the insured bank and deposit insurance company can know the full information and can calculate the default rate of the banks' accurately. We will now discuss the determination of $Q(t)$. Assume that the bank is risk neutral. Different from the theory of credit risk (Ong, 1999), where the values of assets and liabilities of banks is assumed to follow lognormal distribution, we assume that standardized value of the assets and the liabilities of a bank can be described as the stochastic differential equation (1) and (2) which follows Vasicek (1977) model:

$$dr_A = a_A(b_A - r_A)dt + \sigma_A dz_A \tag{1}$$

$$dr_L = a_L(b_L - r_L)dt + \sigma_L dz_L \tag{2}$$

where dzA (dzL) is a standard Wiener process, σ_A (σ_L) is the standard deviation of standardized value of assets (liabilities) of a bank, bA (bL) is the equilibrium standardized value of assets (liabilities) portfolio of long term, $a_A(b_A - r_A)$ (or $a_L(b_L - r_L)$) is the gap between its current value and its long-run equilibrium level and aA (aL) is a parameter measuring the speed at which the gap is closed. Based on Mamon (2004), rA and rB also can be expressed as following stochastic differential equations:

$$dr_A(t) = \mu_A(t)dt + \sigma_A(t)dB_A(t) \tag{3}$$

$$dr_L(t) = \mu_L(t)dt + \sigma_L(t)dB_L(t) \tag{4}$$

where BA(t) and BL(t) are two correlated standard Brownian Motions, and their time-varying correlation coefficient is $\rho(t)$, which can be expressed as

$$\rho(t) = \frac{\sigma_{AL}(t)}{\sigma_A \sigma_L} = \frac{1 - e^{-(a_A + a_L)t}}{a_A + a_L} \tag{5}$$

For proof of equation (5), please see appendix.

$$\mu_A(t) = E(r_A(t)) = e^{-a_A t}(r_A(0) + b_A(e^{a_A t} - 1)) \tag{6}$$

$$\mu_L(t) = E(r_L(t)) = e^{-a_L t}(r_L(0) + b_L(e^{a_L t} - 1)) \tag{7}$$

The dynamically expected default probability of banks can be expressed as follows:

$$\sigma(t) = \sqrt{\sigma_A^2(t) + \sigma_L^2(t) - 2\sigma_{AL}(t)} \tag{8}$$

where

$$\sigma(t) = \sqrt{\sigma_A^2(t) + \sigma_L^2(t) - 2\sigma_{AL}(t)} \tag{9}$$

$$\sigma_{AL}(t) = \frac{\sigma_A \sigma_L}{a_A + a_L}\left(1 - e^{-(a_A + a_L)t}\right) \tag{10}$$

$$\sigma_A^2(t) = \frac{\sigma_A}{2a_A}(1 - e^{-2a_A t}) \tag{11}$$

and

$$\sigma_L^2(t) = \frac{\sigma_L}{2a_L}(1 - e^{-2a_L t}) \tag{12}$$

In what follows, we will use dynamic value at risk (DVaR) and dynamic expected shortfall to estimate the maximum loss at 1 – q confidence level of the bank(s) and use it as the amount of capital which satisfies regulation requirement respectively.

2.1.2. Critical condition of participating deposit insurance instead of raising capital with dynamic value at risk

Define the dynamic value at risk value $DVaR(q,t)$ as total expected loss of the bank(s) at the confidence level of 1 – q and at time t, t $\leq \boldsymbol{T}$ which satisfies:

$$Pr(A(t) - L(t) < -D\,VaR(q,t)) = q \tag{13}$$

where $A(t)$ and $L(t)$ are values of assets and liabilities of bank(s) at time t, t = 1, 2, …, T. Then we have:

$$DVaR(q,t) = \mu_A(t) - \mu_L(t) + z_q\sigma(t) \tag{14}$$

where $\mu_A(t)$ and $\mu_L(t)$ are defined by equations (6) and (7), $\sigma(t)$ is defined by eqaution (10) and z_q is critical value of standard normal distribution $N(0,1)$ with confidence level 1 – q.

Let $r_c(t)$ be the rate of capital cost at time t = 1, 2, …, T, the total capital cost for raising amount of capital, $DVaR(q,t)$ is $DVaR(q,t)\cdot r_c(t)$. Then, the actual amount of capital which the firm should be raising is $DVaR(q,t)(1+r_c(t))$ if accounting for the capital cost.

Assume that if participating in deposit insurance, the net premium rate is Q(t) defined by equation (8) and the volatility of assets and liabilities portfolio of the bank will increase α(t), t = 1, 2, …, T, due to moral hazard created by deposit insurance. Based on the law of large numbers, the net premium paid by the banks for deposit insurance should be equal to the claim loss paid by deposit insurers. However, it holds only if the insured banks have no moral hazard (i.e., additional risk taking enable by insurance protection). In reality, insured banks will take additional risks, if insured, by increasing risks of assets and liabilities. Let DC(t) express the sum of net premium of deposit insurance and increased expected loss at 1 – q confidence level, which results from moral hazard due to deposit insurance, then we have:

$$\begin{aligned} DC(t) = Q(t)\mu_L(t) + \Big(\mu_A(t) - \mu_L(t) + z_q\big(1+\alpha(t)\sigma(t)\big)\Big) - \Big(\mu_A(t) - \mu_L(t) + z_q\sigma(t)\Big)\big(1+r_c(t)\big) = \\ = Q(t)\mu_L(t) - \big(\mu_A(t) - \mu_L(t)\big)r_c(t) - z_q\sigma(t)\big(r_c(t) - \alpha(t)\big) \end{aligned} \tag{15}$$

It is easy to know that the critical condition for participating deposit insurance to instead of raising capital is:

$$DC(t) \leq DVaR(q,t)\cdot r_c(t) \tag{16}$$

That is to say, when the increased expected loss at 1 – q confidential level which results from moral hazard by deposit insurance is less than the total expected loss at 1 – q confidence level if no deposit insurance, participating deposit insurance is more economical than to raise capital, and vice versa.

By combining equations (14), (15) and (16), we obtain the critical value of moral hazard level α*(t) expressed as:

$$\begin{aligned} Q(t)\mu_L(t) - \big(\mu_A(t) - \mu_L(t)\big)r_c(t) - z_q\sigma(t)\big(r_c(t) - \alpha^*(t)\big) \\ = \Big(\mu_A(t) - \mu_L(t) + z_q\sigma(t)\Big)r_c(t) \\ \Rightarrow \alpha^*(t) = 2r_c(t) + \frac{2\big(\mu_A(t) - \mu_L(t)\big)r_c(t) - Q(t)\mu_L(t)}{z_q\sigma(t)} \end{aligned} \tag{17}$$

which means that if risk taking level (the increased percentage of the volatility of asset and liability portfolio) satisfies the condition α*(t) ≤ α*(t), then participating deposit insurance is better than raising capital, and vice versa.

Under ideal condition of completely fair pricing and without moral hazard, we can get the critical rate of capital cost:

$$r_c*(t) = \frac{Q(t)\mu_L(t)}{2\left(z_q\sigma(t) + \mu_A(t) - \mu_L(t)\right)} \tag{18}$$

which means that if the rate of capital cost satisfies the condition $r_c(t) \leq r_c*(t)$, then raising capital is prefferable than participating deposit insurance and vice versa.

2.1.3. Critical condition of participating deposit insurance instead of raising capital with dynamic expected shortfall

Letting DTVaT(q, t) be the expected shortfall at time t at 1 – q confidence level, we have1:

$$DTVaR(q,t) = \mu_A(t) - \mu_L(t) + \frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)} \tag{19}$$

If considering the risk taking after participating deposit insurance, the increased expected shortfall at time t at 1 – q confidence level is

$$\begin{aligned}\Delta\, DTVaR(q,t) \quad &= \left(\mu_A(t) - \mu_L(t) + \frac{\sigma(t)\big(1+\alpha(t)\big)\phi(z_q)}{1-\Phi(z_q)}\right) \\ &\quad - \left(\mu_A(t) - \mu_L(t) + \frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)}\right)\big(1 + r_c(t)\big) \\ &= -\big(\mu_A(t) - \mu_L(t)\big)r_c + \frac{\sigma(t)\phi(z_q)\big(\alpha(t) - r_c(t)\big)}{1-\Phi(z_q)}\end{aligned} \tag{20}$$

Then, the critical condition of participating deposit insurance instead of raising capital is that the net premium plus the increased expected shortfall due to moral hazard by deposit insurance must be equal or less than the capital cost of raising capital, that is:

$$\begin{gathered}Q(t)\mu_L(t) + \Delta\, DTVaR(q,t) \leq DTVaR(q,t)\, r_c \\ \Rightarrow \alpha^*(t) = 2r_c(t) + \frac{2\big(\mu_A(t) - \mu_L(t)\big)r_c(t) - Q(t)\mu_L(t)}{\sigma(t)\phi(z_q)}\Big(1 - \Phi(z_q)\Big)\end{gathered} \tag{21}$$

Similarly, under ideal condition of completely fair pricing and without moral hazard, we can get the critical rate of capital cost as follows:

$$r_c*(t) = \frac{Q(t)\mu_L(t)\sigma(t)}{2\left(\frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)} + \mu_A(t) - \mu_L(t)\right)} \tag{22}$$

2.2. Numerical Analysis with examples

Assume that A(t) and L(t) are the values of assets and liabilities of a bank. The standardized values of A(t) and L(t) are $z_A(t) = \frac{A(t) - min(A(t))}{max(A(t)) - min(A(t))}$. We use the historical financial data of most recent 13 years of six U.S. main banks whose values of assets in most recent year are excess 1.5 billion dollars to illustrate the applications of our method.

1 Please refer to Landsman, Z. and Valdez, E. (2003).

Table 1. Values of the parameters and the asymptotic error of estimation with a Vasicek model of for four U.S. banks.[2]

No. of Banks	a_A	ε_{a_A}	b_A	ε_{b_A}	σ_A	ε_{σ_A}
1	0.4593	0.3404	0.4167	0.2442	0.8440	0.1519
2	0.4248	0.3209	0.4002	0.2659	0.8836	0.1653
3	0.5356	0.3842	0.4337	0.2381	0.9684	0.1945
4	0.4459	0.3328	0.4381	0.3748	0.9358	0.1862
No. of Banks	a_L	ε_{a_L}	b_L	ε_{b_L}	σ_L	ε_{σ_L}
1	0.4607	0.7388	0.4178	0.2430	0.8411	0.1509
2	0.4334	0.7406	0.3919	0.2596	0.8712	0.1610
3	0.5297	0.7254	0.4239	0.2345	0.8703	0.1638
4	0.5602	0.5940	0.3037	0.1799	0.6866	0.1026

Table 2. Critical level of moral hazard with dynamic value at risk at 99.5% confidence level at minimum capital requirement.

$r_c = 0.05$								
t	1	2	3	4	5	6	7	8
No.1	0.0638	0.0651	0.0644	0.0635	0.0628	0.0623	0.0619	0.0617
No.2	0.0226	0.0450	0.0543	0.0591	0.0620	0.0637	0.0648	0.0654
No.3	0.0610	0.0626	0.0621	0.0614	0.0608	0.0605	0.0603	0.0601
No.4	0.0771	0.0768	0.0761	0.0754	0.0750	0.0747	0.0745	0.0744
$r_c = 0.10$								
t	1	2	3	4	5	6	7	8
No.1	0.1651	0.1658	0.1648	0.1637	0.1629	0.1624	0.1620	0.1617
No.2	0.1229	0.1453	0.1545	0.1622	0.1593	0.1639	0.1649	0.1656
No.3	0.1607	0.1626	0.1621	0.1615	0.1610	0.1606	0.1604	0.1603
No.4	0.1831	0.1811	0.1797	0.1787	0.1780	0.1776	0.1773	0.1772
$r_c = 0.15$								
t	1	2	3	4	5	6	7	8
No.1	0.2664	0.2664	0.2652	0.2640	0.2631	0.2624	0.2620	0.2618
No.2	0.2231	0.2455	0.2547	0.2595	0.2623	0.2641	0.2651	0.2658
No.3	0.2605	0.2626	0.2622	0.2616	0.2611	0.2608	0.2606	0.2605
No.4	0.2892	0.2854	0.2832	0.2819	0.2810	0.2805	0.2801	0.2799
$r_c = 0.20$								
t	1	2	3	4	5	6	7	8
No.1	0.3676	0.3671	0.3655	0.3642	0.3632	0.3625	0.3621	0.3618
No.2	0.3234	0.3457	0.3549	0.3597	0.3625	0.3642	0.3653	0.3660
No.3	0.3602	0.3625	0.3623	0.3617	0.3613	0.3610	0.3608	0.3607
No.4	0.3953	0.3897	0.3868	0.3851	0.3840	0.3834	0.3829	0.3827

We name these four banks as No. 1, No. 2, No. 3, No. 4, respectively. Table 1 lists the parameters of Vasicek model with historical data from U,S, Table 2 and 3 list the critical moral hazard level, α*(t) defined as the increased volatilities of asset and liability portfolios of four banks if fully participating deposit insurance with dynamic value at risk at 99.5% confidential level and dynamic expected shortfall at 99% confidential level as minimum capital requirement respectively and when the rate of capital cost takes different values. Table 2 and Table 3 indicate that α*(t) will increase if the rate of capital cost takes great values, meaning that higher capital cost will encourage the insured banks to take more risk if they participate in full deposit insurance. Cenerally speaking, the cost of raising

[2] $\varepsilon_{a_A}, \varepsilon_{b_A}$ and $\varepsilon_{\sigma_A} \varepsilon_{\sigma_i}$ in Table 1 express the error of asymptotic estimation of parameters of a_A. b_A and σ_A while $\varepsilon_{a_L}, \varepsilon_{b_L}$ and ε_{σ_L} in Table 1 express the error of asymptotic estimation of parameters of a_L, b_L and σ_L in the Vasicek model.

capital by banks with lower credit rate will be higher due to their difficulty to raising capital, therefore, the deposit insurance when offered to the with lower credit rate will likely bring about problems caused by moral hazard (i.e., tendency of decision makers with insurance to assume more risk than decision makers without insurance).

Figure 1 and Figure 2 display the change patterns of the critical rate of capital cost with the time for four banks selected when using dynamic value at risk and dynamic expected shortfall as minimum capital requirement respectively. We find from Figure 1 and Figure 2 that the critical rates of capital cost are very small and they are much smaller than the average rate of capital cost for banks3. Therefore, if there is no moral hazard and the deposit insurance is fairly priced, full deposit insurance is more economical and more preferable than raising capital to satisfy the minimum capital requirement but without deposit insurance since capital is an expensive resource.

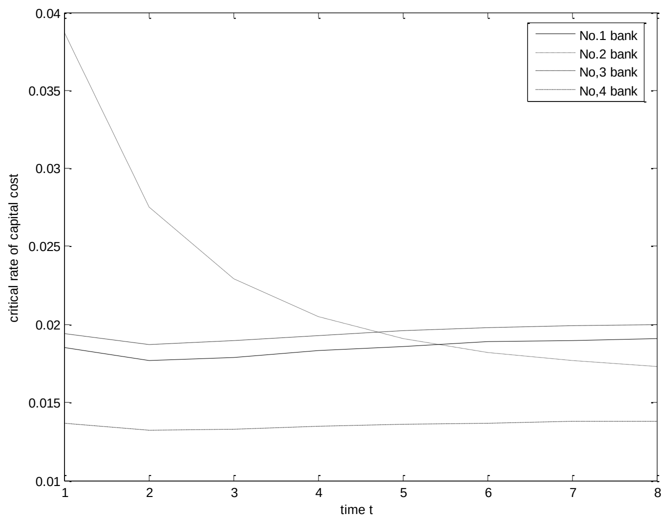

Figure 1. Patterns of change of critical rate of capital cost with time, under criterion of dynamic value at risk. The horizontal axis represents time, and the vertical axis shows the change in cost of capital with time.

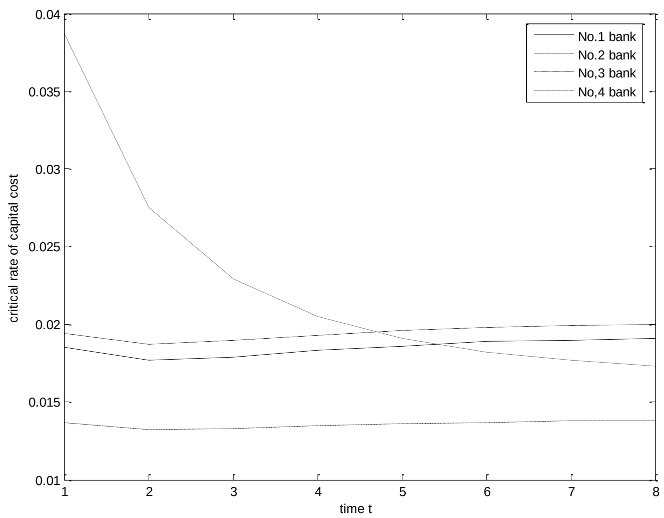

Figure 2. Patterns of change of critical rate of capital cost with time, under criterion of dynamic expected shortfall. The horizontal and vertical axes have analogous meaning to the one presented in Figure 1.

[3] The average rate of capital cost of U.S. banks is 10.4% from 1993-2001 and it is 7.2% from 2002-2009 (Please see Table 1 of King (2009)).

3. Critical conditions that partial deposit insurance and partial capitalization instead of fully raising capital

In the previous section, we discussed the issue under the assumption that the deposit insurance and capitalization can be substituted between each other under certain conditions. However, if banks buy full insurance, there is no incentive for depositors to monitor the banks, the banks will potentially take risks by assuming very aggressive investment strategies, decrease the capital held, and ignore risk management, and the insolvency probability of such insured banks will increase significantly.

Bond and Crocker (1993) point out: " The optimal insurance plan furnishes less than full insurance, which provides depositors with an incentive to require that banks self-protect through capitalization. On the other hand, the current system of full insurance without capitalization is shown to provide no incentive to self-protect, generating increased bank risk through de-capitalization". In this section, we will discuss how to determine the critical condition of partial deposit insurance and partial capitalization to instead of fully raising capital if we use less than full insurance in order to avoid moral hazard and if we set proportion of deposit insurance at different levels.

Assume that the proportion of deposit insurance is p(t), where p(t) < 1, t = 1, 2, …, T, and increased moral hazard level is represented by increased volatility of the value of asset and liability portfolio expressed as α1(t). Other assumptions are same as those in section 1, the previous section. In what follows, we will discuss the critical conditions with partial insurance.

3.1. With criterion of DVaR (q,t) as minimum capital requirement

Let the total expected loss be DC1(t), which is the sum of net premium of partial deposit insurance, the capital cost for raising capital for expected loss with 1 – q confidence level for the remaining part of no deposit insurance and the increased expected loss with 1 – q confidence level resulting from moral hazard due to deposit insurance. Then we have:

$$\begin{aligned} DC_1(t) &= Q(t)\mu_L(t)p(t) + \big(1-p(t)\big)\Big(\mu_A(t)-\mu_L(t)+z_q\big(1+\alpha_1(t)\big)\sigma(t)\Big)r_c(t) \\ &+\Big(\mu_A(t)-\mu_L(t)+z_q\big(1+\alpha_1(t)\big)\sigma(t)-\Big(\mu_A(t)-\mu_L(t)+z_q\sigma(t)\Big)\big(1+r_c(t)\big)\Big)p(t) \\ &= Q(t)\mu_L(t)p(t)-2\big(\mu_A(t)-\mu_L(t)\big)r_c(t)p(t) \\ &+z_q\sigma(t)\alpha_1(t)\big(p(t)+r_c(t)\big)-3z_q\sigma(t)r_c(t)p(t) \end{aligned} \tag{23}$$

The critical condition of partial deposit insurance and partial capitalization rather than fully raising capital is:

$$\begin{aligned} &DC_1(t) \le DVaR(q,t)\,r_c(t) \\ &\Rightarrow \alpha_1^*(t) = \frac{2\Big(\mu_A(t)-\mu_A(t)+z_q\sigma(t)\Big)p(t)r_c(t)-Q(t)\mu_L(t)p(t)}{z_q\sigma(t)\big(p(t)+r_c(t)(p(t)-1)\big)} \end{aligned} \tag{24}$$

We note that equation (24) will be simplified into equation (17) when p(t) = 1.

3.2. With criterion of DTVaR (q,t) as minimum capital requirement

If the proportion of deposit insurance at time t is p(t), then, the critical condition of participating partial deposit insurance, partially raising capital instead of completely raising capital is that the sum of net premium of partial deposit insurance, the capital cost of raising capital for expected shortfall with 1 - q confidence level for the remaining part of no deposit insurance and the increased expected shortfall due to moral hazard by partial deposit insurance must be equal or less than the capital cost of completely raising capital if no deposit insurance, that is:

$$Q(t)\mu_L(t) + D_1 TVaR(q,t)\, r_c(t)p(t) + \Delta D_1 TVaR(q,t) \leq DTVaR(q,t)\, r_c$$
$$\Rightarrow \alpha_1^*(t) = \frac{2\left(\mu_A(t) - \mu_A(t) + \frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)}\right)p(t)r_c(t) - Q(t)\mu_L(t)p(t)}{\frac{\phi(z_q)}{\left(1-\Phi(z_q)\right)\sigma(t)\left(p(t) + r_c(t)(p(t)-1)\right)}} \tag{25}$$

where

$$D_1 TVaR(q,t) = \mu_A(t) - \mu_L(t) + \frac{\sigma(t)\left(1+\alpha_1(t)\right)\phi(z_q)}{1-\Phi(z_q)} \tag{26}$$

$$DTVaR(q,t) = \mu_A(t) - \mu_L(t) + \frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)} \tag{27}$$

and

$$\begin{aligned} \Delta D_1 TVaR(q,t) \quad &= \left(\mu_A(t) - \mu_L(t) + \frac{\sigma(t)\left(1+\alpha_1(t)\right)\phi(z_q)}{1-\Phi(z_q)}\right) \\ &\quad - \left(\mu_A(t) - \mu_L(t) + \frac{\sigma(t)\phi(z_q)}{1-\Phi(z_q)}\right)\left(1 + r_c(t)\right) \\ &= -\left(\mu_A(t) - \mu_L(t)\right)r_c + \frac{\sigma(t)\phi(z_q)\left(\alpha_1(t) - r_c(t)\right)}{1-\Phi(z_q)} \end{aligned} \tag{28}$$

If p(t) = 1, equation (28) will be simplified as equation (21).

3.3. Numerical analysis

In what follows, we will use No.1 bank as an example to do numerical analysis. Figure 3 and Figure 4 describe the relationship between the proportion of deposit insurance and critical level of moral hazard with time using dynamic risk at value at 99.5% confidential level as minimum capital requirement (Please note that as Table 2 in above section shows that the results if using dynamic expected short fall at 99.0% confidential level as minimum capital requirement is similar, we will neglect the detailed analysis on it) . We set the rate of capital cost at 0.10 and 0.20 respectively, and for other values of parameters please see Table 1. Figure 5 and Figure 6 display the relationship between the proportion of deposit insurance and the capital amount with criterion of dynamic value at risk as minimum capital requirement with time. Figure 3 and Figure 4 indicate that the higher the proportion of deposit insurance, the lower the critical level of moral hazard it is allowed. Since partial deposit insurance will encourage depositors to require that banks self-protect through capitalization, the capability for banks to resist risks will enhance and more risk taking is allowed. Figure 5 and Figure 6 shows that the dynamic value at risk at 99.5% level will decreases with the increase of the proportion of deposit insurance, which means that more deposit insurance will encourage insured banks to hold less capital.

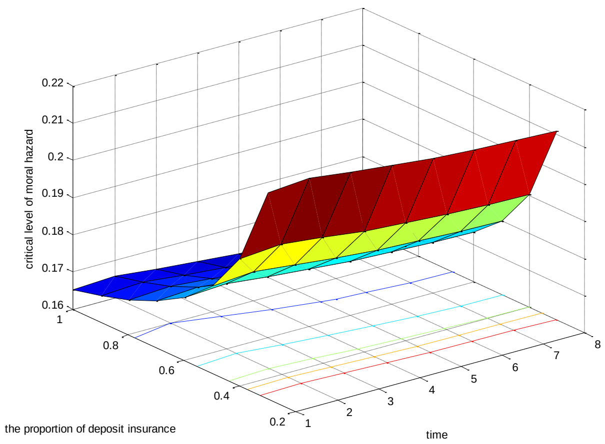

Figure 3. Patterns of change of critical level of moral hazard for different proportion of deposit insurance and with change in time (r_C = 0.10).

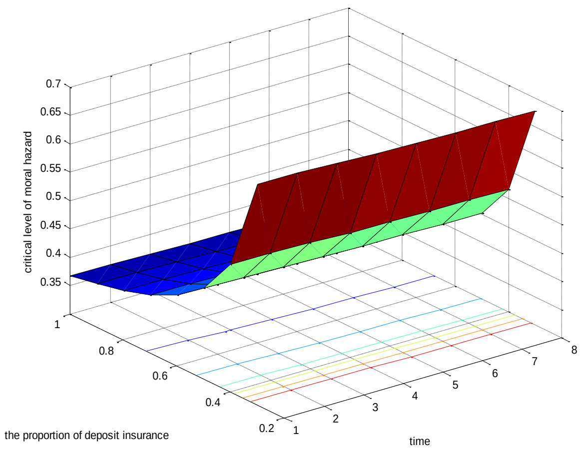

Figure 4. Patterns of change of critical level of moral hazard for different proportion of deposit insurance and with change in time (r_C = 0.20).

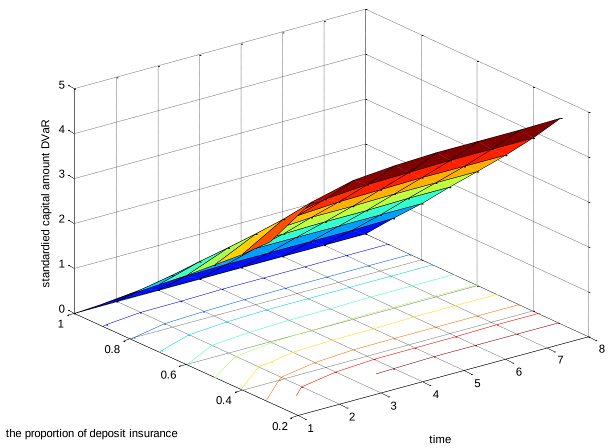

Figure 5. Patterns of change of standardized dynamic value at risk for different proportion of deposit insurance and with change in time (r_C = 0.10).

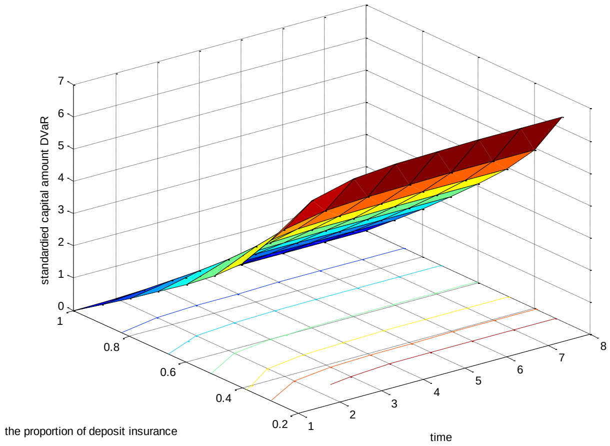

Figure 6. Patterns of change of standardized dynamic value at risk for different proportion of deposit insurance and with change in time (r_C = 0.20).

4. Conclusions

In this article, we discuss whether deposit insurance is complementary or substituted with capitalization. We discuss the condition(s) under which deposit insurance and capitalization can be substituted for each other, under the assumption of existence of moral hazard. We establish the models to determine the critical bound of raising capital, as opposed to obtaining deposit insurance. We also discuss the regulatory capital level of banks if using capitalization as an alternative to deposit insurance. The results of our analysis show that the critical level of moral hazard increases with the increase in capital cost, and it decreases with the increase of proportion of deposit insurance, which means that lower proportion of deposit insurance allows higher level moral hazard due to higher level of capitalization incentive by depositors. It is important for banks to strengthen risk management so as to decrease the risk of the volatility of assets' value. The further study will focus on considering the effects of the systemic risks, the effects of information asymmetric on the determination of the default rate of the insured banks.

It should be noted, and indeed stressed, that existing regulatory risk-based capital requirements in both banking and insurance are generally imposed based on models describing the company itself, without reference to deposit insurance or guaranty funds available from the governments for protection of customers' funds. On the other hand, regulators impose risk-based capital requirements primarily because of concern for excessive moral hazard created by deposit insurance, tacitly admitting the relationship we study in our work. Our work suggests that regulatory regimes for financial intermediaries may need to be examined in a way that acknowledges the complementary nature of capital and deposit insurance. We believe future empirical examination of that complementary interaction, in a global context, under varying deposit insurance schemes, may be a very valuable contribution to research on the crucial topic of effective prudential regulation.

Funding Statement

This research received no external funding.

Acknowledgment

Acknowledgments to anonymous referees' comments and editor's effort.

Author contributions

Conceptualization: Hong Mao, Krzysztof Ostaszewski, Jin Wang; Investigation: Hong Mao, Krzysztof Ostaszewski, Jin Wang; Methodology: Hong Mao, Krzysztof Ostaszewski, Jin Wang; Formal analysis: Hong Mao, Krzysztof Ostaszewski, Jin Wang; Writing – original draft: Hong Mao; Writing – revisions, review & editing: Hong Mao, Krzysztof Ostaszewski, Jin Wang.

Conflict of interest

The authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

References

Bhattacharya, S., Boot, A., and Thakor, A. (1998). The economics of bank and regulation. *Journal of Money, Credit and Banking*, 30, 745-770. https://doi.org/10.2307/2601127

Bond, E. W., and Crocker, K.J. (1993). Bank capitalization, deposit insurance and risk categorization. *Journal of Risk and Insurance*, 60, 543-569. https://doi.org/10.2307/253379

Demirgüc-Kunt, A., and Detragiache, E. (2002). Does deposit insurance increase banking system stability? *Journal of Monetary Economics*, 49, 1373-1406. https://doi.org/10.1016/S0304-3932(02)00171-X

Demirgüç-Kunt, A., Kane, E., and Laeven, L. (2015). Deposit insurance around the world: A comprehensive analysis and database. *Journal of Financial Stability*. https://doi.org/10.1016/j.jfs.2015.08.005

Diamond, D., and Dybvig, P. H. (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy*, 91, 401-41. https://doi.org/10.1086/261155

Diamond, D., and Rajan, R. (2000). A theory of bank capital. *Journal of Finance*, 55, 2431-65. https://doi.org/10.1111/0022-1082.00296

Diamond, D. W., Kashyap, A. K., and Rajan, R. G. (2017). Banking and the Evolving Objectives of Bank Regulation. *Journal of Political Economy*. https://doi.org/10.1086/694622

Fama, E. (1985). What's different about banks. *Journal of Monetary Economic*, 15, 29-39. https://doi.org/10.1016/0304-3932(85)90051-0

Fungacova, Well, and Zhou. (2010). Bank capital, liquidity creation and deposit insurance. *Working paper*, 1-29. https://doi.org/10.1007/s10693-016-0240-7

James, C. (1987). Some evidence on the uniqueness of bank loans. *Journal of Financial Economics*, 19, 217-235. https://doi.org/10.1016/0304-405X(87)90003-1

Kantas, G. (1986). Deposit insurance and the discount window: Pricing under asymmetric information. *The Journal of Finance*, XLI, 437-450. https://doi.org/10.1111/j.1540-6261.1986.tb05047.x

King, M. R. (2009). The cost of equity for global banks: A CAPM perspective from 1990 to 2009. *BIS Quarterly Review*, September 2009. https://ssrn.com/abstract=1472988

Landsman, Z., and Valdez, E. (2003). Tail Conditional Expectation for Elliptical Distributions. *North American Actuarial Journal*, 7, 55-118. https://doi.org/10.1080/10920277.2003.10596118

Liu, H., Li, R., and Yuan, J. (2018). Deposit insurance pricing under GARCH. *Finance Research Letters*. https://doi.org/10.1016/j.frl.2018.02.013

Mao, H., and Cheng, J. (2020). Optimal capitalization and deposit insurance strategies with regard to moral hazard. *Journal of Economics and Business*, 108, 1-11. https://doi.org/10.1016/j.jeconbus.2019.105885

Matutes, C., and Vives, X. (1996). Competition for deposit, fragility, and Insurance. *Journal of Financial Intermediation*, 5, 184-216. https://doi.org/10.1006/jfin.1996.0010

Merton, R. (1977). Analytic derivation of the cost of deposit insurance and loan guarantees: An application of modern option pricing theory. *Journal of Banking and Finance,* 1, 3-11. https://doi.org/10.1016/0378-4266(77)90015-2

Ong, M.K. (1999). Internal Credit Risk Models: Capital Allocation and Performance Measurement. *Risk Books*, London. http://ndl.ethernet.edu.et/bitstream/123456789/19836/2/88.%20%20Michael%20K.%20Ong.pdf

Pennacchi, G.G. (2005). Risk-based capital standard, deposit insurance, and pro-cyclicality. *Journal of Financial Intermediation*, 14, 432-465. https://doi.org/10.1016/j.jfi.2004.09.001

Santos, J. A. C, 2001, Bank Capital Regulation in Contemporary Banking Theory: A Review of the Literature, *Financial Markets, Institutions and Instruments*. https://doi.org/10.1111/1468-0416.00042

Santos, J., 2006, Insuring banks against liquidity shocks: The role of deposit insurance and lending of last resort, *Journal of Economic Survey*, 20: 459-482. https://doi.org/10.1111/j.0950-0804.2006.00286.x

Bastos, P., Silva, J., Verhoogen, E. (2018). Export Destinations and Input Prices. *American Economic Review* 108, 353-392. https://doi.org/10.1257/aer.20140647

Cantoni, E., and Pons, V. (2022). Does Context Outweigh Individual Characteristics in Driving Voting Behavior? Evidence from Relocations within the United States. *American Economic Review* 112, 1226-1272. https://doi.org/10.1257/aer.20201660

Head, K., and Mayer, T. (2019). Brands in Motion: How Frictions Shape Multinational Production. *American Economic Review* 109, 3073-3124. https://doi.org/10.1257/aer20161345