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In search of an optimal public policy in a pandemic: The question of lives versus livelihood

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ABSTRACT

The paper addresses the alternative policy options available to address the question of lives versus livelihood in an SIRD model augmented with a macroeconomic structure. An important contribution of the paper lies in designing the policy of lockdown dependent on the extent of the constraint on the health facilities. The paper supplements the literature with a less stringent version of the lockdown policy, viz. soft lockdown policy which is shown to be more attractive from a public policy standpoint and has actually been practised in many countries across the globe during the recent pandemic. Finally, the optimal policy derived on the basis of the level of lockdown and adjustment of the binding constraint on health facilities depends on the objective of policy makers contingent on the relative weights of lives versus livelihood.

KEYWORDS

COVID-19; Corona virus; Macro-SIRD Model; Hard Lockdown; Soft Lockdown

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1. Introduction

The objective of the present paper has been evaluation of the policy of direct intervention vis-a´-vis other less stringent measures to arrest the spread of an infectious disease such as the Covid- 19 pandemic that rocked the world in recent time. Using an aggregative framework the paper incorporates the evolution of the infected population from asymptomatic to mild and then to severe depending on the nature of required treatment and highlights the role of binding health infrastructure. In order to arrest the initial spread of the disease and to prepare for medical intervention majority of the countries initially imposed all economic activities except medical and some limited emergency services and all other activities that involve meeting other people-hard lockdown and then subsequently eased the stringent measures depending on the severity of the spread-partial lifting the lockdown from different sectors, micro lockdown strategies etc. The latter has been called soft lockdown. This mix of strategies has been practiced in majority of the countries. China is the country that followed a very stringent lockdown policy with a aim of zero covid case. A notable exception is Taiwan which followed a policy of encouraging extensive use of mask, minimal meeting etc. Sweden also did not implement lockdown policy. As is expected strict lockdown strategy leads to fall in aggregate output and employment and other kinds of misery to general population and more so for the poor. At the same time no lockdown policy also leads to spread of the disease leading to deaths. So there is a wedge between lives versus livelihood. As for India real life data shows that India experienced several peaks of COVID-19 cases since the outbreak of the pandemic in early 2020. The first peak occurred in September 2020, when the country recorded a daily average of more than 90,000 cases, during this time the death rate was 2.3% (for 1st September, 2020). The second wave hit India in March 2021 and was more severe than the first wave, with the number of cases rising sharply from less than 20,000 to over 400,000 daily cases in May 2021, during this time the death rate was 1.35% (for 15st May, 2021). During this wave, India also recorded a high number of deaths, with the daily death toll reaching over 4,000 at its peak. The third wave started in October 2021, with daily cases increasing from less than 20,000 to around 100,000 in November 2021, during this time the death rate was 1.38% (for 20st November, 2020). However, this wave was milder than the previous waves and did not result in a significant increase in deaths. The fourth wave has now begun according to the Ministry of Health and Family Welfare, Government of India (https://www.mohfw.gov.in/). The growth rate of GDP for first quarter of 2020-21 over the previous quarter fell by 3.6% with lockdown in force in India during the quarters of 2020-21 (based on data provided by Government of India, 2020).

Our objective in this paper has been to find an optimal path that can be followed to achieve a combination of minimum number of deaths with minimum loss of aggregate output measured by gross domestic product. Model simulations for plausible parameter values with a single mutation of the virus show that a policy of no lockdown finds the pandemic to end after around 630 days, when herd immunity is reached, but with a substantial loss of lives as well as more than 1% fall in the aggregate output. If, however, a strict lockdown policy is implemented as and when the health constraint binds, the loss of lives is only about 0.77% for the same period as the no lockdown case, but the economic loss is about 5% and it takes much longer time to reach herd immunity. A soft lockdown policy which is less stringent than a hard lockdown that lifts the extent of the lockdown depending on how far the health infrastructure binds produces outcomes in between these two extremes in terms of both loss of lives and economic loss measured by the fall in gross domestic product or decrease in employment. Before we proceed further it may be noted that this model should not be considered for forecasting purpose, rather it aims to explore the evolution of the disease over time and its control under alternative policy regime. The parameter values have been chosen keeping a country like India in mind. The parameters of relevance, such as rate of infection and its severity, death rate and other factors such as demography, health infrastructure etc. determine the evolution of the disease which has been found to differ across countries (please see Agarwal et al, 2022 for a comparison of India and other countries and Bhattacharjee et al, 2022, for states of USA). However, parameters of our model can be amended to characterize any other country with different institutional structure that includes demographic structure, population density, rural-urban divide etc. along with health infrastructure, social and economic structure.

There has been a spurt in the volume of the published papers and work in progress in the field of economics with the onset of Covid-19. This literature employs what can be called macro-SIRD (Susceptible Infection Recovered Dead) model that supplements the epidemiology laws governing the spread of the virus with a set of additional equations to reflect the economic behaviour in a representative agent framework. The laws of epidemiology draw on the classic model of epidemiology (Kermack and McKendrick, 1927), or its more recent vintage (Hethcote, 2000; Chowell et al., 2009). A few important epidemiology models that ad- dress specifically Covid-19 pandemic are (Anastassopoulou et al., 2020; Bertozzi et al., 2020; Sameni, 2020). A good survey on the literature of economic issues of Covid-19 can be found in Border et al. (2021). The spread of the virus gives a negative shock to the supply of labour as infection starts rising leading to death of a large population. This in turn transmits negative shocks to aggregate production, consumption etc. and the subsequent rounds leading to what has often been called pandemic led recession (Gregory et al., 2020; Guerrieri et al., 2020; Kaplan et al., 2020). It can so happen that with mixed supply and demand shocks the recovery can be V-, U- or W-shaped or can even get stuck in a sustained bad state with an L-shape (Sharma et al., 2021). The process continues until a cure, usually a vaccine becomes available or when the herd immunity is reached transforming the epidemic or the pandemic into an endemic phenomenon.

The justification for intervention policy stems from the fact that there is a negative externality in an epidemic (and more so in a pandemic) as uninfected individuals become susceptible to the disease when they come in contact with infected individuals in the work place or any other gathering (Bryant and Elofsson, 2021; Dimdore-Miles and Miles, 2020; Farboodi et al., 2020). Thus the need for public action has been advocated in the literature in various forms. A number of studies suggests that containment of infected people so as to reduce the rate of infection (Berger et al., 2020; Bethune and Korinek, 2020; Gatto et al., 2020; Grigorieva et al., 2020). An alternative policy option advocates containment measure in the form of consumption tax and a lump sum transfer (Eichenbaum et al., 2020). The former reduces consumption and also makes leisure more attractive so that mixing of people both for purchase of goods and supply of labour decreases leading to a fall in the infection rate. The tax is rebated to households so that disposable income remains unchanged. More direct policy to arrest the spread of the virus has been proposed in the form of social distancing (Jones et al., 2020). The most favoured intervention policy as it suspends economic activities and other gatherings that has been advocated is direct lockdown (Alvarez et al., 2021; Aspri et al., 2021; Caulkins et al., 2021; Casseli et al., 2021). Instead of a general lockdown policy Gori et al. (2021) strongly argue a publicly funded policy of targeting case based testing, tracing and isolation.

Lockdown policy to arrest spread of infection has a wedge between loss of lives versus loss of livelihood. Combining labour market data with simulation of an agent based model for Chile Fosco and Zurita (2021) shows that the policy of lockdown in place in the first five months of the pandemic led to a 8% loss in output in the short run and 56% reduction in infection. In a counterfactual exercise a reduction of 92% in infection is achieved with a reduction of 10.5% in output. Hence, the case for fiscal stimulus has been suggested during the lockdown period to compensate for economic loss (Kaplan et al., 2020). In the presence of heterogenous agents the case for policy of sequential lifting of lockdown has also been advocated (Rampini, 2020). However, Born et al. (2020) suspects the efficacy of lockdown policy. Using age specific demographic profile of population et al. (2021) shows that a targeted lockdown has a lower cost in terms of lower GDP as well as lower fatality rate, while Gollier (2020) advocates a policy of differential lockdown policy for the old as opposed to the young and the middle aged. It is further reported that different strategies can achieve the same goal when number of patients exceeds health care facilities (Caulkins et al., 2020).

The upshot of the above discussion is that the issue pertaining to the intervention policy for arresting the spread of the Covid-19 pandemic is far from a point of convergence. The most advocated policy of direct lockdown has been questioned on many grounds, in particular on the issue of lives versus livelihood. The contribution of the present paper in the literature is in respect of a new policy paradigm that which is in between the standard practice of the policy of no lockdown and hard lockdown. It is named soft lockdown. Optimisation of the objective function is conducted with constraints on the health infra structure for both health service providers and hospital beds. Simulation exercises show that the proposed soft lockdown policy achieves an intermediate outcome in terms of the contraction in GDP and death from disease. It is further shown that the options available to the policy makers are not independent of the objective of the policy makers – a stringent penalty on the deviation of the actuals from the targets favours a policy of no lockdown while the policy of strict lockdown is favoured for a less stringent objective in respect of deviation of actuals from the target. With this introduction we organize rest of the paper into 3 sections–Section 2 proposes the model, Section 3 the simulation results and discussion and the last section concludes with scope of future work.

2. Model

The model in this paper follows a standard SIRD model for the evolution of the disease dynamics, but gives an important role to the asymptomatic population for the spread of the disease as in Basak et al. (2021). The justification stems from empirical studies (Al-Qahtani et al., 2021). The model employs a more disaggregated framework for the symptomatic patients with two kinds of infections, viz. mild infection and severe infection. In the former case the patients do not need hospitalisation while in the second case hospitalisation is very much required. Hence, two crucial factors that are very important in our analysis for reducing the death rate are the availability of doctors including other health workers for the patients with mild infection and both doctors (including health workers) and hospital beds for treating the severely infected patients. It has been reported that rapid escalation of number of infections led to the scarcity of health care facilities including doctors, hospital beds, availability of oxygen etc. thereby affecting patient outcomes (Ji et al, 2020; Moolla & Hiilamo, 2023). With the rise in the number of either or both of the mild or severely infected patients to a very high level available doctors cannot provide treatment and/ or hospital beds become unavailable. In this situation imposition of lockdown becomes imminent to arrest the number of infected patients. Imposition of lockdown can also be justified to reduce the mixing of population.

The total population is assumed to remain fixed at N during the period of analysis, but there are two kinds of susceptible population – general, $S_{g,t}$ and health workers, $S_{h,t}$ at any t; the latter includes both doctors and other health workers (equations (1) and (2)). It is assumed that there are three stages of infection-asymptomatic, mild and severe. In the beginning the infected population is assumed to remain asymptomatic (A_t) for a few days (equation on (3)), then some of them recover, and the rest, $I_{m,t}$ starts showing mild symptoms equation (4). They need treatment in the out patient department or telephonic advice, but do not require hospitalisation. Some of them receive treatment and the rest do not depending on whether enough doctors are available or not. In either case a proportion of the mildly infected population becomes severely ill ($I_{c,t}$) and the rest recovers with a higher probability if treatment received than without as in equation (5) and (6) respectively. The severely ill pool of patients receive treatment and the rest do not receive treatment. In this case availability of treatment can be constrained by either of the availability of doctors or the availability of hospital beds. Total number of death is given by equation (7). At any point of time total population, N is distributed in different types of equations (8).

The evolution of the disease dynamics described above are presented in terms of the following equations.

$$\Delta S_{g,t+1} = -\lambda_{g,t}\, S_{g,t}\frac{A_t}{N} \tag{1}$$

$$\Delta S_{h,t+1} = -\frac{\lambda_{h,t}S_{h,t}\left(\lambda_{0,t}\, A_t + I_{c,t} + \lambda_{1,t}I_{m,t}\right)}{N} \tag{2}$$

$$\Delta A_{t+1} = \frac{\lambda_{g,t}S_{g,t}A_t}{N} + \frac{\lambda_{h,t}S_{h,t}\left(\lambda_{0,t}A_t + \lambda_{1,t}I_{m,t} + I_{c,t}\right)}{N} - \beta_0\alpha_0 A_t - \beta_0(1-\alpha_0)A_t \tag{3}$$

$$\begin{aligned}\Delta I_{m,t+1} = \beta_0\alpha_0 A_t - \beta_1\left(\alpha_{m,t}(1-\alpha_{22}) + \left(1-\alpha_{m,t}\right)(1-\alpha_1)\right)I_{m,t}\\ -\beta_1\left(1-\alpha_{m,t}(1-\alpha_{22}) - \left(1-\alpha_{m,t}\right)(1-\alpha_1)\right)I_{m,t}\end{aligned} \tag{4}$$

$$\begin{aligned}\Delta I_{c,t+1} = \beta_1\left(1-\alpha_{m,t}(1-\alpha_{22}) - \left(1-\alpha_{m,t}\right)(1-\alpha_1)\right)I_{m,t}\\ -\gamma_1\left(\alpha_{c,t}(1-\alpha_{42}) + \left(1-\alpha_{c,t}\right)(1-\alpha_3)\right)I_{c,t}\\ -\gamma_1\left(1-\alpha_{c,t}(1-\alpha_{42}) - \left(1-\alpha_{c,t}\right)(1-\alpha_3)\right)I_{c,t}\end{aligned} \tag{5}$$

$$\begin{aligned}\Delta R_{t+1} = \beta_0(1-\alpha_0)A_t + \beta_1\left(\alpha_{m,t}(1-\alpha_{22}) + \left(1-\alpha_{m,t}\right)(1-\alpha_1)\right)I_{m,t}\\ +\gamma_1\left(\alpha_{c,t}(1-\alpha_{42}) + \left(1-\alpha_{c,t}\right)(1-\alpha_3)\right)I_{c,t}\end{aligned} \tag{6}$$

$$\Delta D_{t+1} = \gamma_1\left(1-\alpha_{c,t}(1-\alpha_{42}) - \left(1-\alpha_{c,t}\right)(1-\alpha_3)\right)I_{c,t} \tag{7}$$

$$\left(S_{g,t} + S_{h,t}\right) + A_t + \left(I_{m,t} + I_{c,t}\right) + R_t + D_t = N \tag{8}$$

where, $\lambda_{g,t}$ = rate of change of susceptible general population to asymptomatic population over time when mixing with the asymptomatic population, $\lambda_{h,t}$=rate of change of susceptible health workers to asymptomatic population over time when mixing with the critical patients being treated and proportion of mild patients being treated and a proportion of asymptomatic population, $\lambda_{0,t}$=proportion of asymptomatic population who come in contact with the health care workers, $\lambda_{1,t}$=proportion of the mildly infected population under treatment who come in contact with the health care workers, β_0=rate of depletion from the pool of asymptomatic population, α_0=probability that an asymptomatic person gets mildly infected (as opposed to getting re- covered), $\alpha_{m,t}$=probability that a mildly infected patient can not receive treatment at time t, β_1=rate of depletion from the mildly infected population under treatment, α_1=probability that mildly infected patient under treatment falling critically ill, α_{22}=probability that a mildly infected untreated patient falling critically ill, $\alpha_{c,t}$=probability that a critically ill patient does not get treatment at time t, α_{42}=probability of death of untreated critically ill patients, γ_1 = rate of depletion from the treated critically ill population, α_3=probability of death of treated critically ill population.

The disease dynamics is represented in terms of a flow chart in Figure 1.

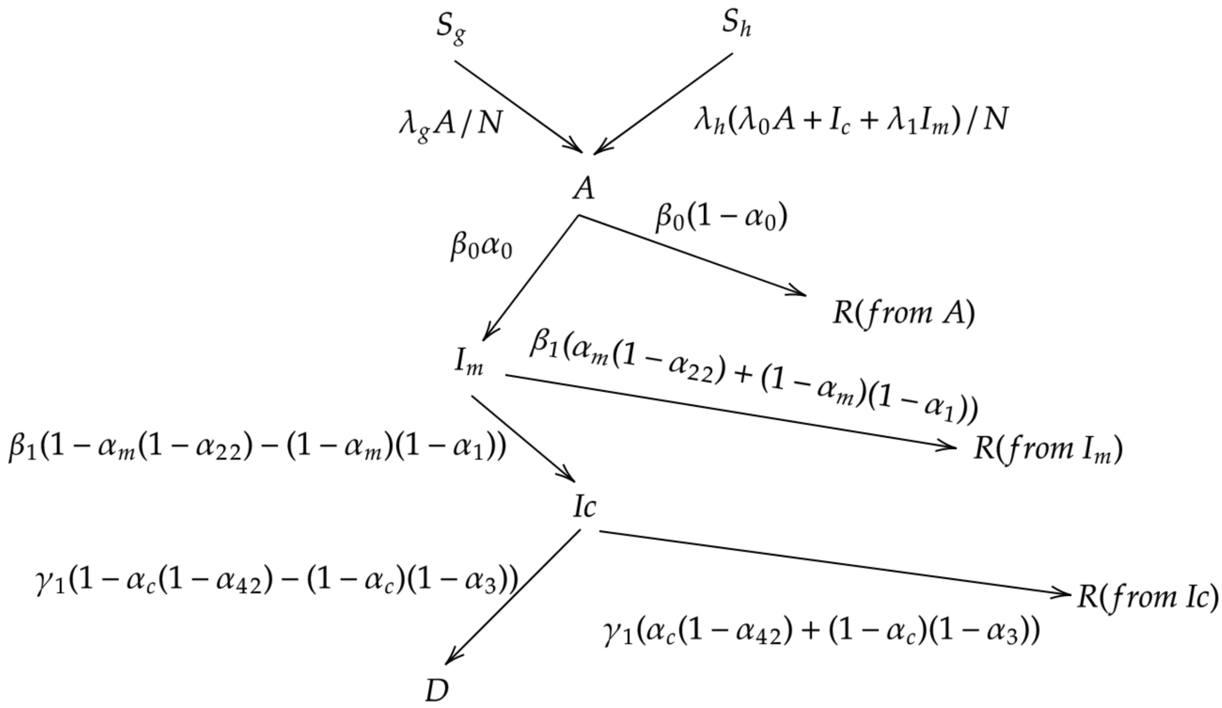

Figure. 1. Flowchart from susceptible to recovery or death.

The flowchart above shows the disease dynamics from the components in one stage to the next depending upon the probability. E.g., the asymptomatic population contributes to the change in the mildly infected population according to probability ($\beta_0\alpha_0$)-the first component in equation (4) and recover by $\beta_0(1-\alpha_0)$-the first component in equation (6). The negative components in equation (4) are those who are removed from mildly infected pool. Similarly for other equations. It may be noted that $\lambda_{g,t}$ is the outcome of two parameters, viz. rate of mixing of the population, c_m, and rate of infection of the disease, c_i. These two can be combined to generate $\lambda_{g,t}$ as in below:

$$\lambda_{g,t} = c_m c_i \theta_t^{1+\nu} \tag{9}$$

where ν is a parameter and θ_t is lockdown parameter, such that 0<θ_t<1. c_m, c_i are dependent on country characteristics, such as population density, rural-urban distribution of the population etc. Similarly for the λ_h term. With no lockdown θ_t takes the value of unity so that the rate of transmission, $\lambda_{g,t}$ has full impact. With θ_t<1, which happens when a policy of lockdown is implemented, the rate of transmission $\lambda_{g,t}$ gets reduced. The rates of change, viz. $\lambda_{g,t}$, $\lambda_{h,t}$, $\lambda_{0,t}$, $\lambda_{1,t}$, and the probability terms (transition probabilities), viz. $\alpha_{m,t}$ and $\alpha_{c,t}$, $\alpha_{41,t}$ are indexed by time implying that these vary with time. The rates of change terms vary depending upon whether there is lockdown or not and if there is lockdown then type of lockdown, viz. hard or soft. The probability of getting treatment vary over time because of the availability of doctors and beds which in turn are dependent on policy opted for lockdown.

Denoting the number of available doctors for each of the mild and severely infected patients (doctor-patient ratio) by $\phi_{m,t}$ and $\phi_{c,t}$ respectively on each day and total beds available for treatment by B_t the transition probabilities are of receiving treatment are defined as in below:

$$\alpha_{m,t} = 1 - \frac{exp\left(-\frac{\lambda_m}{(1-min(h_{g,t},1))}\right)}{exp(-\lambda_m)} \tag{10}$$

and

$$\alpha_{c,t} = 1 - \frac{\exp\left(-\max\left(\lambda_c/(1-min(h_{g,t},1)), \lambda_b/(1-min(h_{b,t},1)\right)\right)}{exp(-max(\lambda_c,\lambda_b))} \tag{11}$$

where $h_{b,t} = \frac{I_{c,t}}{B_t}, h_{g,t} = \frac{(I_{m,t}\phi_{m,t} + I_{c,t}\phi_{c,t})}{(S_{h,t} + R_{h,t-\delta_h} + A_{h,t})}$, with $\phi_{m,t} = \phi_{m,t}^n$ (if there is no lockdown), $\phi_{m,t} = \phi_{m,t}^l$ (if there is lockdown) and $\phi_{c,t} = \phi_{c,t}^n$ (if there is no lockdown), $\phi_{c,t} = \phi_{c,t}^l$ (if there is lockdown), $(S_{h,t} + R_{h,t} - \delta_h + A_{h,t})$=number of available doctors at time t. The number of beds, B_t at time t is assumed constant in this model, but in reality during the period of stress on health facilities other facilities are converted into beds/infrastructure. In the equation (10), we have a dynamic probability $\alpha_{m,t}$ which depends on the availability of the doctors per mildly or critically infected individual with respect to the section of susceptible, recovered and the asymptotic health-care group. If the ratio $h_{g,t}$ is less than 1, the probability $\alpha_{m,t}$ is adjusted likewise, and when the ratio $h_{g,t}$ exceeds 1 implying there is no availability of the health-care group, and hence the probability $\alpha_{m,t}$ becomes 1. Similarly the dynamic probability $\alpha_{c,t}$ depends on the availability of the doctors and the hospital beds for the critically infected individuals. The mechanism for the $\alpha_{c,t}$ works similarly as of $\alpha_{m,t}$ except for there is another term for the hospital beds, $h_{b,t}$

For the sake of simplicity the services of the doctors and other health workers have been clubbed together in the same category. Separate constraints on the availability of each of the other health workers can be introduced in a more generalized and real life model. However, the constraint on the availability of doctors can capture other health workers also when there is a fixed proportion of requirement between doctors and other health workers. The availability of beds can also be thought of as a composite good that includes other peripherals like facilities of ICU, availability of oxygen and other medicines etc. In a situation when the disease spread reaches very high level, either of the availability of doctors or that of the beds binds. However, the available data shows that it is the number of beds that becomes most critical for the treatment of the patients in the Indian context and elsewhere too (Buonsenso et al, 2021; Robeznieks, 2022; The Mint, 2021).

Total population in this model is treated as the total working population; no distinction is made between working and non-working (i.e. old and child) for the sake of simplicity. In other words demographic characteristic of the population is not taken into considered. Needless to mention it is an abstraction from reality. The implications of the demographic features of the population are discussed later. The labour force at any time point t is the total population less the infected and dead plus recovered with the necessary delay to return to work, which may be said to be the adjusted sum of S_g and S_h (see (12)). However, the recovered patients face delay of a few days to join labour force. In this respect we are making a distinction in respect of labour force engaged in health services and other economic activities. The former is not subject to any lockdown. So when lockdown is imposed, it affects only the general population engaged in general economic activities. It may be noted that a distinction is made between labour-output ratio in the health sector from the labour-output ratio in the general economic activities. Accordingly the labour force is derived from (1)-(8) to be:

$$L_t(\theta_t) = \theta_t\left(S_{g,t} + A_{g,t} + R_{g,(t-\delta g)}\right) + a\left(S_{h,t} + A_{h,t} + R_{h,(t-\delta h)}\right) \tag{12}$$

where a is a multiplier used for the conversion of the output of a health worker to that of a general worker, δ_h and δ_g are the delay time for the corresponding worker to get back to the work and θ_t is the lockdown factor, $\varepsilon(0,1)$. A value of $\theta_t = 1$ implies no lockdown in force and all of the available labour force is allowed to work while imposition of lockdown means $\theta_t < 1$ so that only a part of the labour force is allowed to work. There is a one-to-one relation between labour and total output given by equation (18). Given the available labour as given by equation (12) total production of goods and services is given by equation (19). Equation (18) is amended to yield equation (19) to capture lockdown and disease related factors. A low value for actual labour force available (given by equation (19)) will lead to low output. This in turn means unemployment, lower production and consumption. If the output is very low then it can lead to loss and bankruptcy. The aggregate production function has both the general goods and services and health services as components.

The policy instrument for imposition of lockdown in this paper is linked with the availability of doctors and hospital beds (vis-a´-vis infrastructure). As and when either or both the constraints bind or close to the binding level lockdown is imposed via restriction on the use of labour. It remains in place so long as the constraint is relaxed to a reasonably low level. This is called hard lockdown in this paper. A certain minimum level of essential acitivities, such as shopping for food articles, electricity, gas, banking, health etc. are allowed even under hard lockdown. The lockdown function takes two values and is defined at each t as in below:

$$\begin{aligned} \theta_t &= \theta_0 \; if \; h \geq 1 \\ &= 1 \; if \; l < 1 \end{aligned} \tag{13}$$

where θ_0 is the level of activities allowed under lockdown including modes such as work from home. It also includes social distancing, and h and l refer to the maximum and normal levels of of stress that health facilities can accommodate defined as in below:

$$h = Min\left(S_{h,t} + A_{h,t} + R_{h,t}, B\right) \tag{14}$$

$$l = Min\left(\frac{2}{3}B_t, \frac{S_{h,t} + A_{h,t} + R_{h,t}}{\sqrt{\frac{\phi_{m,t}^n \times \phi_{c,t}^n}{\phi_{m,t}^l \times \phi_{c,t}^l}}}\right) \tag{15}$$

The normal stress level, l is the minimum of the two terms in the numerator adjusted by the geometric mean of the two types of stress levels (with and without lockdown). These forms of h and l are among the alternative policy rules, other forms can also be tried.

There is another option for the lockdown policy, viz. the degree of lockdown is set to be determined by a rule depending on the extent of the constraints on the availability of doctors and/or hospital beds. So, θ_t is now replaced by a time dependent function. This policy of lockdown has been called *soft* lockdown in this paper. The rule for changing the extent of lockdown can mainly be thought of three kinds, viz. (a) a constant proportion of the deviation of actual availability from the binding level, called linear, (b) an increasing function of the deviation from binding level, called convex, and (c) a decreasing function of the deviation from binding level, called concave, defined as in below:

$$\text{(a) } \theta_t = \theta_0 \frac{\kappa - l}{h - l} + \frac{h - \kappa}{h - l} = (\theta_0 - 1)\frac{\kappa - l}{h - l} + 1$$

$$\text{(b) } \theta_t = (\theta_0 - 1) f_{conv}\left(\frac{\kappa - l}{h - l}\right) + 1$$

$$\text{(c) } \theta_t = (\theta_0 - 1) f_{conc}\left(\frac{\kappa - l}{h - l}\right) + 1 \tag{16}$$

where f_{conv}, f_{conc} are convex and concave increasing bijections from [0, 1] $\mapsto$ [0, 1], respectively and

$$\kappa = Min\left(\frac{I_{c,t}}{1/\phi_{c,t}^n} + \frac{I_{m,t}}{1/\phi_{m,t}^n}, \frac{3}{2} I_{c,t}\right) \tag{17}$$

A power function of the form Z^{μ} with $0 < \mu < 1$ represents a concave adjustment rule for θ, with μ>1 represents a convex adjustment rule and μ=1 represents a linear adjustment. A value of μ equal to zero represents hard lockdown with θ equaling θ_0.

The economic activities, measured by GDP in this model, is represented by an aggregate Neo-classical production function of Cobb-Douglas variety with constant returns to scale in two factors employed at time t - capital, K_t and labour L_t and a technology parameter V. With capital, K_t and the technology parameter, V remaining constant in short period of time the product $VK_t^{1-\alpha}$ is normalized at unity. Thus aggregate output, Y_t is given by,

$$Y_t = L_t^{\alpha} \tag{18}$$

where $0 < \alpha < 1$. Any variation in aggregate output is obtained by varying labour. The benchmark full employment level output, $\bar{Y}$ is the output corresponding to full employment of labour $\bar{L}$ before the pandemic began, i.e. N. It is expected that actual output during the pandemic period is well below $\bar{L}$ level, even when there is no lockdown, because population growth is ignored in the model while the labour force decreases due to death. In this model total population is considered to be the working population or the labour force in the age group 15 to 64. That is, the population, N, is equated with the entire labour force for the sake of simplicity. This age group is around 65% of total population in India.

We do not distinguish between different components of GDP for two reasons. First, unlike Kaplan et al. (2020), Bethune and Korinek (2020), Eichenbaum et al. (2020), Hall et al. (2020) we do not aim to find the optimal lockdown policy as a central planner's welfare maximisation problem. Instead, the optimum lockdown in this model is obtained by minimizing a loss function defined in terms of deviation of aggregate output and number of deaths from pre-specified targets, viz. pre-lockdown levels. This approach does not need any distinction between components of aggregate output, all that matters is the aggregate output. Secondly, unlike Jones et al. (2020) and others, we do not distinguish between meeting of people on the basis of consumption and production purposes.

When lockdown (hard or soft lockdown) is in place the aggregate output is not given by (18), instead it is amended to incorporate the lockdown policy parameter θ_t with a policy of lockdown (either of the hard or soft) in place as in below:

$$Y_{l,t} = \left(L_t(\theta_t)\right)^{\alpha} \tag{19}$$

Finally, the loss function, Ψ for the evaluation of the policy performance is defined in terms of the deviation of GDP, Y_t from a target level, $\bar{Y}$ and total fatality, D_t from a target level, which in this case is assumed to be zero over the usual death from other diseases. As these two items are not conformable for addition we use a weight, χ which represents the statistical value of life. The objective function for the policy makers is given by

$$\Psi_t = \sum_{j=0}^{t_0}\left[(1 - \frac{Y_{l,t+j}(\theta_t)}{\bar{Y}}\bar{Y})^m + \chi(D_{t+j}(\theta_t))^m\right] \tag{20}$$

where, t_0 is the terminal period for which the policy of lockdown is implemented. The control of the minimization exercise is θ_t, the type and extent of lockdown under alternative policy regimes and $Y_{l,t}$ and D_t are both state variables. The parameter m is the power of the loss function. For $m = 1$, it is linear in target output and number of deaths, though convex in the control θ_t and μ. For $m = 2$, it becomes the standard quadratic loss function in target output and number of deaths. The minimisation of Ψ by choice of the control θ and μ implies the choice for a given m.

This form of the objective function has important implications. When the policy measures fails to reach the targets the convex ($m > 1$) loss function imposes severe penalty than when the policy is adopted based on a concave ($m < 1$) loss function, whereas when the targets are achieved (or exceeds) the benefit is lower than that obtained from a concave loss function. In case of the linear loss function there is no asymmetry in respect of penalty *vis*-a-*vis* benefit. It is to be noted that whatever be the form of the policy function depending upon the value of m, it is nonlinear in the health policy rule for relaxing (or tightening) the health facilities, μ and the degree of imposing lockdown θ. These two are the relevant policy variables for the control of the output stabilisation and the spread of the disease (or more specifically number of deaths). The spread and control of a pandemic in a short horizon problem, in the specific case of Covid-19 is expected to continue for 3 years if Spanish flu is an indicator. Hence the objective function is not discounted. Statistical value of life χ is drawn from the standard literature defined as an estimate of the financial value that society places on reducing the average number of deaths by one (Schelling, 1968). Employing the widely adopted method of Viscusi and Aldy (2003) the statistical value of life is estimated to be around USD 0.43 million per death at current prices in India. A more liberal estimate is provided by Majumder and Madheswaran (2018) to the tune of USD 0.64 million for the Indian population. This estimate is used in the present paper.

Thus the problem of the policy maker is given by:

$$\min_{\theta_t} \Psi_t \tag{21}$$

s.t. (1) to (8), (10), (11), (13) (or (16) depending upon lockdown regime type), (19).

This completes the description of the model economy with the disease dynamics. In the next section when we undertake the simulation of the model we adopt two approaches to explore the policy options for the control of the spread of infection. First, we will take up a general characterization of the intertemporal trajectory with or without lockdown policy. It lists different levels of aggregate output and number of deaths with or without lockdown policy. Second, we provide the optimal bundle of aggregate output and number of deaths obtained via optimization of the loss function.

3. Simulation results

The simulation is undertaken with plausible parameter and initial values to reflect on the disease dynamics over time that match India. The complete simulation is done using Python 3.6. The parameter values and initial values are provided in Tables 1-2. The rates and probabilities, viz. λ 's, α 's and β 's are calculated using available data from governmental and international sources (www.mohfw.gov.in, www.worldometers.info/coronavirus etc.). These are further compared with the values used in other studies for India and other countries to arrive at reasonable and meaningful estimates. The parameter of the aggregate production function, α is the share of wages in GDP, which is obtained from ILO (2018). As already explained in the previous section the model in this paper considers the working population at the age group 15 to 64, which in the Indian case constitutes around 65% of total population of 1.37 Billion. The number of health workers (doctors in this paper) is obtained from the World Development Indicators (WDI) database of the World Bank duly adjusted for the 65% of the population. The number of hospital beds is also obtained from the WDI database duly adjusted in the same way.

Table 1. Parameter values.

Parameter	S_g	S_h	A	α_h	B	β_0	α_0	β_1	α_1
Value	884e6	0.76e6	1000	3	0.49e6	0.01	0.4	0.13	0.1
Parameter	α_{22}	α_{42}	γ_1	α_3	λ_m	λ_c	λ_b	α	c_m
Value	0.3	0.1	0.06	0.01	2	2	2	0.35	0.02
Parameter	c_i	ν	δ_h	δ_g	δ_a	δ_m	δ_i	χ	$\bar{Y}$
Value	6	0.2	14 days	14 days	3 days	5 days	14 days	0.64e6	2.7e12

Table 2. Parameter Values with respect to the state of lockdown.

Parameters	No Lockdown	Lockdown
λ_0	0.7	0.2
λ_1	0.6	0.5
$\phi_{m,t}$	1/15	1/26
$\phi_{c,t}$	1/7	1/10

As for the parameters $\phi_{m,t}$, $\phi_{c,t}$ in (10) and (11) we consider reasonable values in the two cases when there is no lockdown and when there is lockdown. In the former case the health system is under normal level of operation with a lower probability of not getting treatment while in the later case the health system is stressed because of a very high level of the spread of the disease with too many patients and the probability of not getting treatment rises. The form of the probability functions (10) and (11) ensures that with rising number of patients there is stress on the health system and the probabilities of no treatment increases. We further introduce a mutation of the virus after 450th day, i.e. after five quarters from the time of occurrence of the disease. This is introduced by an exogenous increase of 40% in the rate of transmission of infection, ci for the general population as well as among the health workers. This leads to an increase of $\lambda_{g,t}$ and $\lambda_{h,t}$ from 0.12 to 0.168 for the general population and from 0.08 to 0.1 for the health workers respectively in the absence of lockdown (i.e. $\theta_t = 1$). However, it may be noted that the value of $\lambda_{g,t}$ and $\lambda_{h,t}$ decreases when lockdown (hard or soft) remains veffective, because during lockdown there is restriction on the mixing of the population given by θ_t and hence the disease spreads at a lower rate. It may be noted that vaccination of the population can be thought of as a special case of mutation when the mutation of the virus takes a form that reduces rates of infection, severity and death. However, the trajectory will depend on the coverage of the vaccine programme. One can make the mutation an inverse function of vaccination status. Thus $\lambda_{g,t}$ and $\lambda_{h,t}$ are functions of state of vaccination.

First, we consider the case in the absence of any lockdown policy. In this case the virus spread is allowed to take place without any intervention in the form of lockdown. The total number of infections (of the twin categories), recovery and death and proportion of fatality for the general population as well as for the health workers (measured in terms of observed recoveries) are provided in Table 3 at the end of each quarter. The evolution of the disease is also presented in Figure 2-6. The disease spreads at a high rate until the third quarter, there after slows down, reaches its peak between 4th and 5th quarter, shows sign of decline, but again increases with the occurrence of the new variant with higher rate of infection (λ_g and λ_h) after 5th quarter (450th day). However, it reaches the peak soon, at around 500th day, and then starts declining. It is evident from the figures as well as the tables that the spread of the disease stabilizes after 630th day, i.e. after around 7th quarter with the infection at 53.46% of the population. Thence it becomes endemic with total death a little more than 5.29 million which is 0.598% of the population.

Table 3. Evolution of the disease: No lockdown.

Day	$A_0+\sum\Delta A$	$\sum\Delta I_m$	$\sum\Delta I_c$	R_T	R_0	D	$D/(D+R_0)$	$D_h/(D_h+R_h)$	$Y_t/\bar{Y}$
90	0.068578	0.020557	0.001581	0.045884	0.01505	0.000009	0.00059765	0.00042	0.9999969
180	0.908478	0.278246	0.02221	0.627009	2.537447	0.000163	0.00006423	0.00304	0.9999787
270	10.931416	3.381367	0.314639	7.609496	2.537447	0.007974	0.00313268	0.00103	0.9998258
360	92.196396	30.615438	6.193251	68.679211	22.756055	0.341618	0.01479015	0.00480	0.9987061
450	234.530714	89.542325	23.478961	215.229483	80.915997	2.077223	0.02502883	0.00946	0.9963413
540	425.07921	163.130137	44.666874	392.573195	147.87799	3.979567	0.02620592	0.00990	0.9942628
630	472.967767	188.720005	53.157648	466.006075	182.926069	5.287181	0.02809144	0.01113	0.9937873
720	475.458153	190.163371	53.373361	470.081254	184.83901	5.295764	0.02785268	0.01116	0.9942521
810	475.568953	190.227948	53.382197	470.269311	184.930624	5.295952	0.02784023	0.01116	0.9946557

Note: (i) The level variables are in Million. (ii) Active cases as on 810th day is 0.001372 Million and fatality rate for the whole *period (upto 810th day) is 2.784%. (iii) Death as % of population on 630th (810th) day is 0.598% (0.599%). (iv) Herd immunity as % of population on 630th (810th) day is 53.46% (53.75%).*

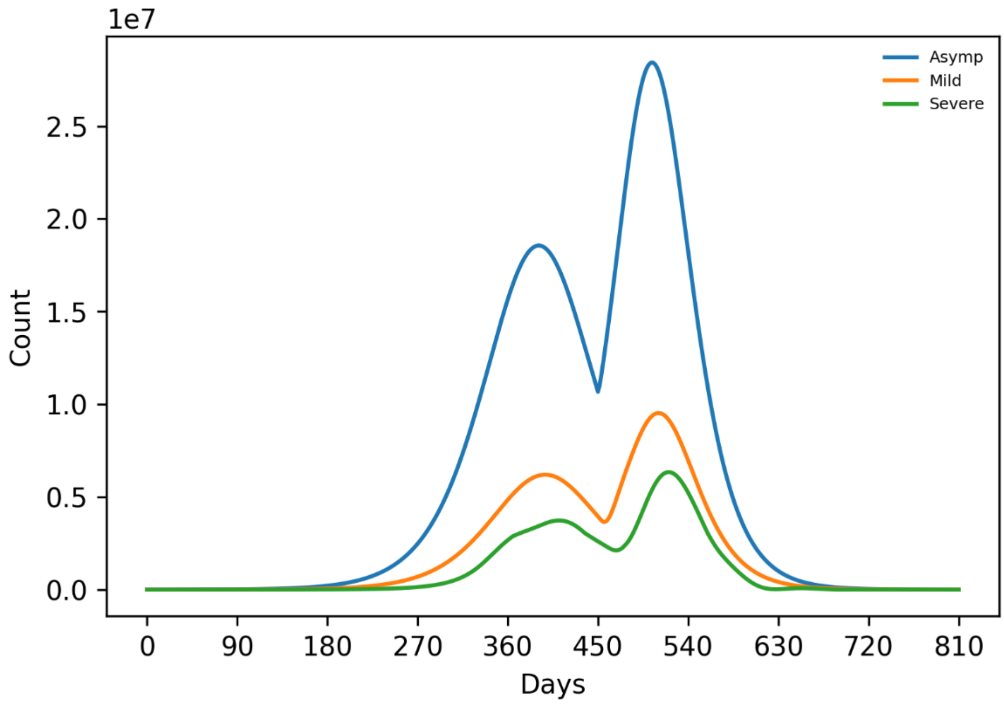

Figure 2. Evolution of Disease for Asymptomatic, Mild & Severe-No lockdown.

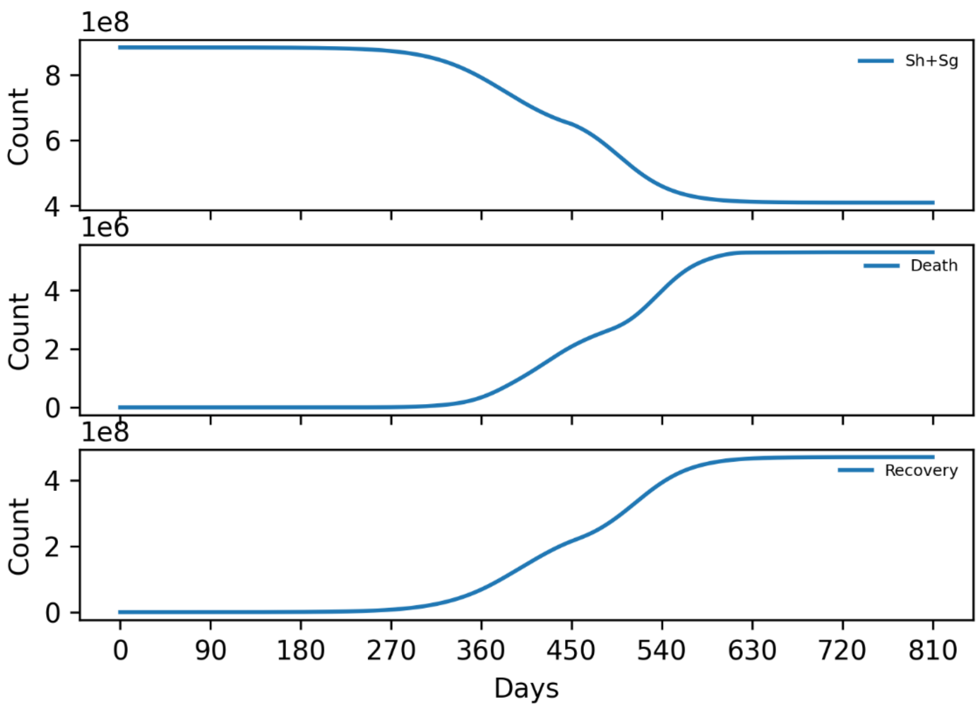

Figure 3. Susceptible, Death and Recovery-No lockdown.

The fatality rate is 2.81% as on the 630^{th} day. The herd immunity is calculated as total asymptomatic cases as percentage of starting population. It may be noted that the herd im- munity is reached with the spread of the disease among 53.6% of the population which is well below the expected value of above 60% as the forecasting models of the epidemiology literature predicts. It so happens in this model because we have considered only one type of population with the same rate of mixing. The rate of mixing differs among working population and nonworking population and the children. Hence a more realistic model with realistic demography will take care of this apparent anomaly.

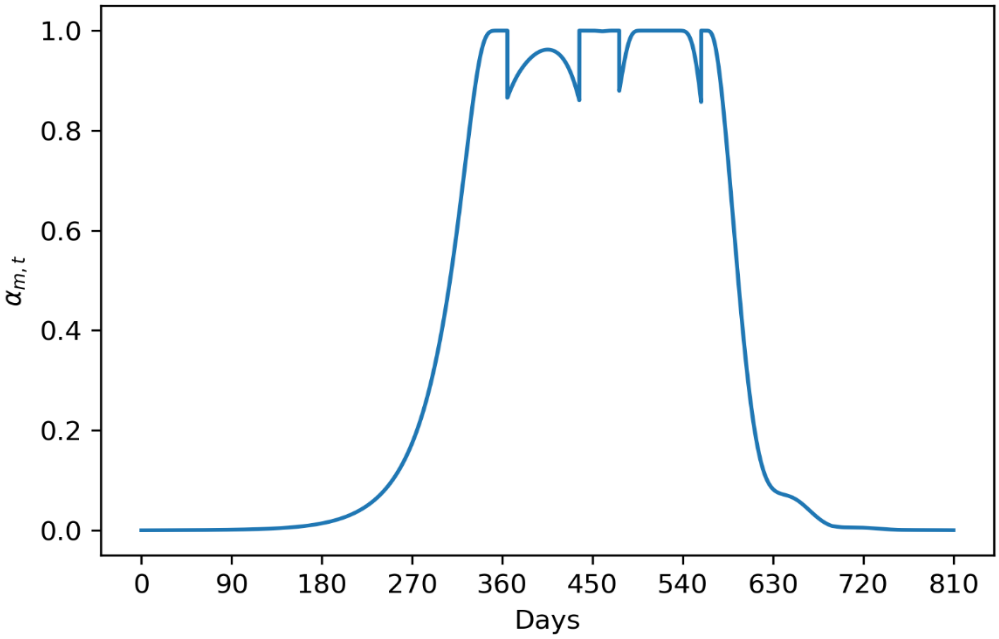

Figure 4. Probability of no treatment for mild infection-No lockdown.

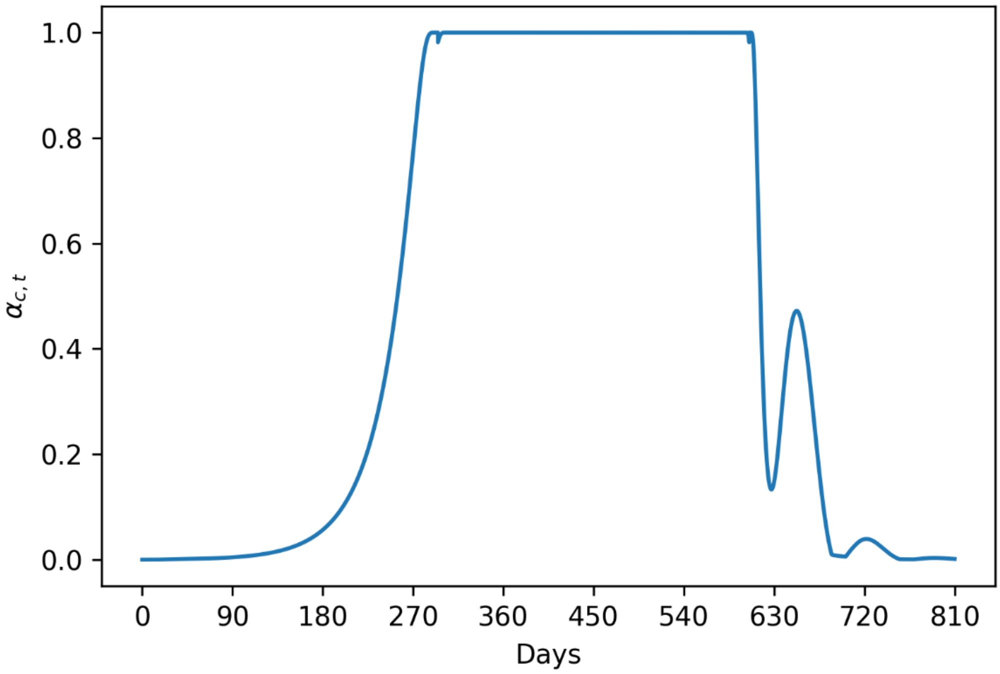

Figure 5. Probability of no treatment for severe infection-No lockdown.

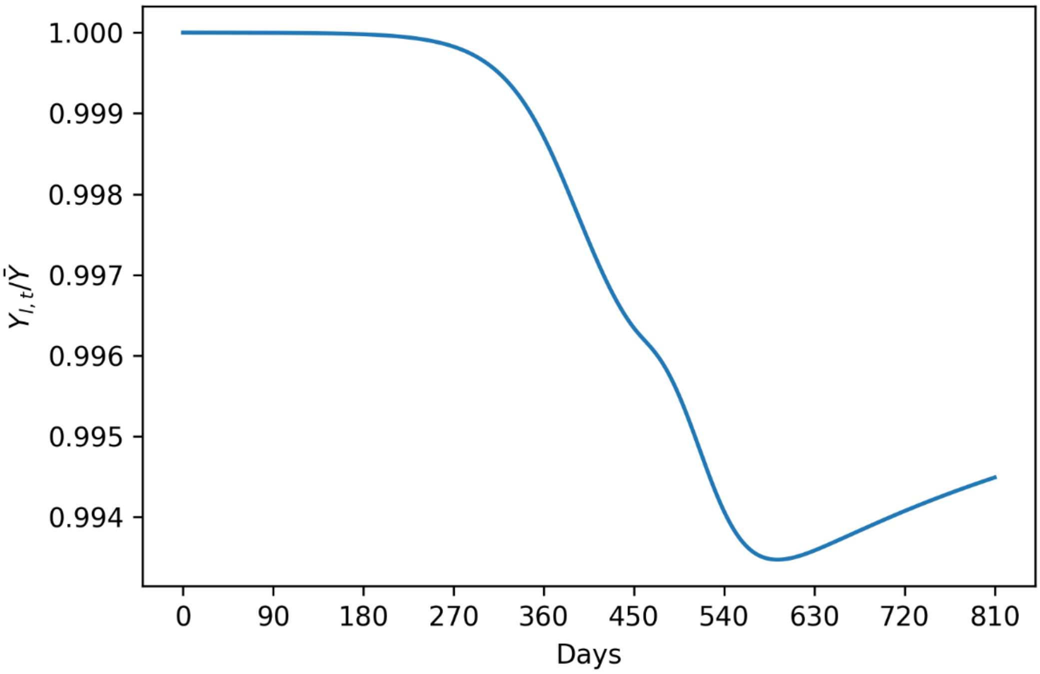

Figure 6. Actual to full employment output-No lockdown.

The probability of no treatment for both the mild as well as severe infections rapidly rises with the spread of infection from the 3^{rd} quarter, which attains very close to unity immediately. The probabilities in both the cases remain at unity until the infection starts declining after around 500^{th} day. However, the probability of no treatment for the mild case shows a few small dips. Once herd immunity is reached there is no significant change in the total number of asymptomatic, mild or severe cases of infection or the number of death. If another mutation with higher infection rate occurs then the date of stabilization would further extend but may not be too far, because with rising infection of the population number of cases will increase sufficiently to reach the level of herd immunity. The GDP falls about 1% in this case reaching the lowest point on 630^{th} day, there after starts recovering. The pace of recovery is slower than the fall reflected in a lower (absolute value) slope of the former than the latter.

This case is hypothetical as the no lockdown policy was not in force in India or elsewhere across the globe, but it is a benchmark for comparing different policy options. However, the relevance of this case stems from the fact that it provides a comparison with the Spanish flu of 1918-20. The Spanish flu continued for about three years but the cases of death occurred mainly in the first year across the globe with 1.42% of population in the first year and 2.1% for the three years period taken together. The corresponding rates were 4.1% and 5.22% respectively for India which was the second highest being next to Kenya (Barro et al., 2020). The three year aggregate rate for European countries and USA were well below less than 1%. The medical infrastructure at that time was much less by today's standard and for India it was almost non- existent for the native population. Though quarantine was implemented in India and elsewhere, no large scale lockdown was imposed. So largely it is the herd immunity that arrested the spread of the disease.

Next we consider the results of the simulation with a policy of hard lockdown presented in Table 4 and in Figure 7-11. As was discussed earlier hard lockdown is implemented by the rule given by (13), i.e. as and when either of the demand for doctors or hospital beds exceeds the availability and it is lifted once the constraint is relaxed. Accordingly the hard lockdown policy is implemented (lifted) on the 306^{th} (336^{th}), 463^{th} (504^{th}), 551^{th} (593^{th}), 645^{th} (684^{th}), 744^{th} (782^{th}) day. We have shown the case of hard lockdown policy with $\theta_t = 0.5$, i.e. allowing 50% workforce. Of the total workforce 30% comprises of the employment in the essential services, such as health care, transport etc., the rest from the remaining 70%. It is evident from Figure 7 that as the disease starts rising very fast after 3^{rd} quarter as in the no lockdown case leading to constraint on the health services. Then the policy of lockdown is imposed from 306^{th} day. This reduces the mixing of the population and hence the value of λ_g (and also λ_h) leading to a relaxation of the constraint as given by (13). The lockdown is then withdrawn after about a month on the 336^{th} day. Withdrawal of lockdown allows the spread of the disease at the previous rate of λ_g until it reaches a level for (13) to bind and the process of imposition of lockdown repeats itself. With the arrest of the spread of the disease temporarily during the period of lockdown the hospital services gets better off but it as well reduces the process of herd immunity and hence the steady state. In fact only 13.7% of the population attains immunity on the 630th day and 20% on 810th day in this case. The herd immunity is reached well after 2200 days (not shown here in the figure). The total number of infections as well as fatalities is much lower with hard lockdown until 7th or 9th quarter. The total cases of fatality on the 630th and 810th day are 0.77% and 0.122% of the population respectively.

Table 4. Evolution of the disease: Hard lockdown

Day	$A_0+\sum\Delta A$	$\sum\Delta I_m$	$\sum\Delta I_c$	R_T	R_0	D	$D/(D+R_0)$	$D_h/(D_h+R_h)$	$Y_t/\bar{Y}$
90	0.068578	0.020557	0.001581	0.045884	0.01505	0.000009	0.0005845	0.00042	0.999997
180	0.908478	0.278246	0.02221	0.627009	0.209641	0.000163	0.0007769	0.00304	0.999979
270	10.931416	3.381367	0.314639	7.609496	2.537447	0.007974	0.0031327	0.00103	0.999826
360	31.996303	12.629799	1.929382	31.002286	12.28231	0.158609	0.0127490	0.00465	0.981699
450	51.978179	19.211336	2.692120	46.045757	17.45348	0.195524	0.0110785	0.00397	0.985141
540	86.833713	32.399672	5.089192	78.427825	30.45174	0.417297	0.0135183	0.00481	0.971024
630	121.22110	47.087237	8.195643	114.546675	45.19171	0.679914	0.0148221	0.00509	0.960488
720	150.13404	59.627045	10.707362	145.630954	57.97404	0.895595	0.0152132	0.00518	0.953281
810	176.98216	71.098143	12.878879	174.085632	69.63349	1.082866	0.0153128	0.00522	0.948260

Note: (i) The level variables are in Million. (ii) Lockdown imposed (lifted) on 306th (336th), 463th (504th), 551th (593th), 645th (684th), 744th (782th) days. (iii) Active cases as on 810th day is 0.381787 million and the fatality rate for the whole period (upto 810th day) is 1.5313%. (iv) Death as % of population on 630th (810th) day is 0. 077% (0.122%). (v) Herd immunity as % of population on 630th (810th) day is 13.7% (20.0%).

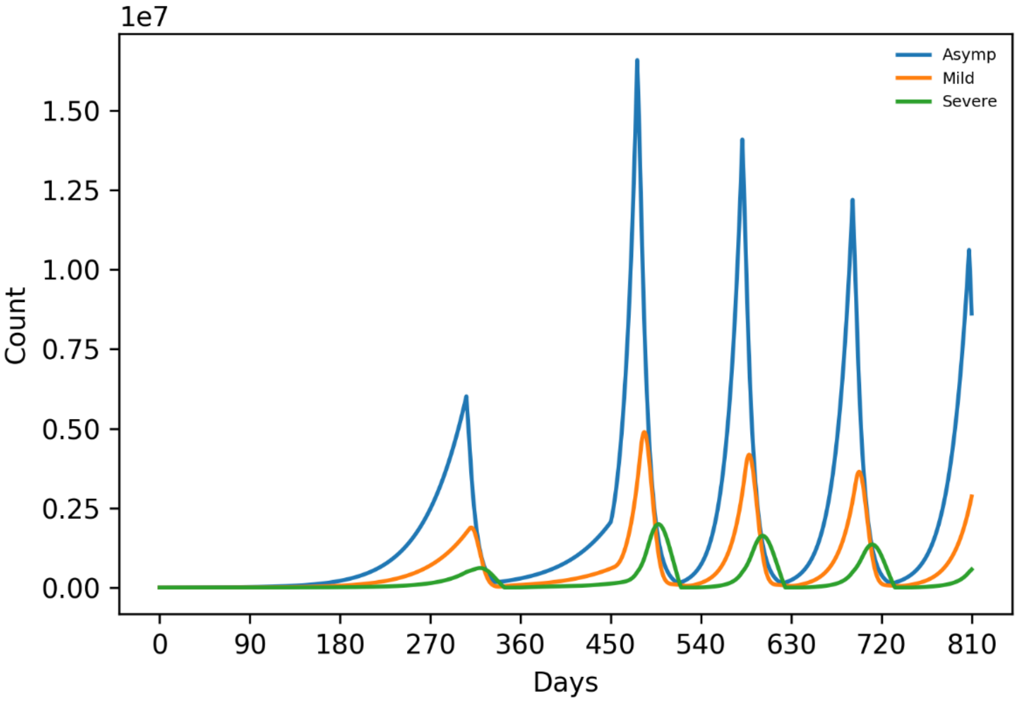

Figure 7. Evolution of Disease for Asymptomatic, Mild & Severe-Hard lockdown.

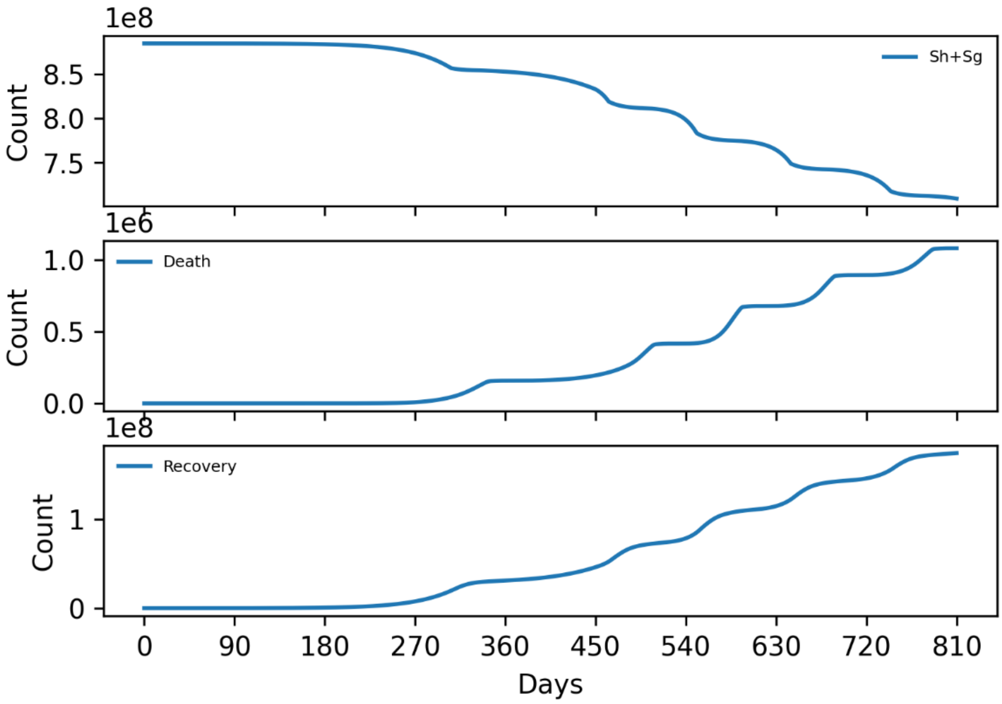

Figure 8. Susceptible, Death and Recovery-Hard lockdown.

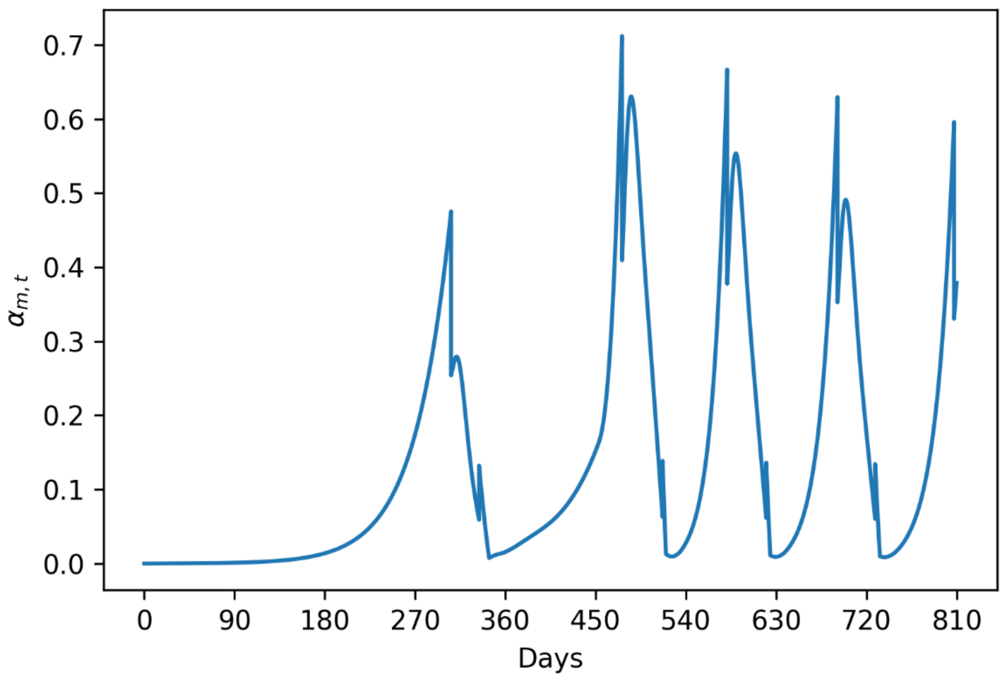

Figure 9. Probability of no treatment for mild infection - Hard lockdown

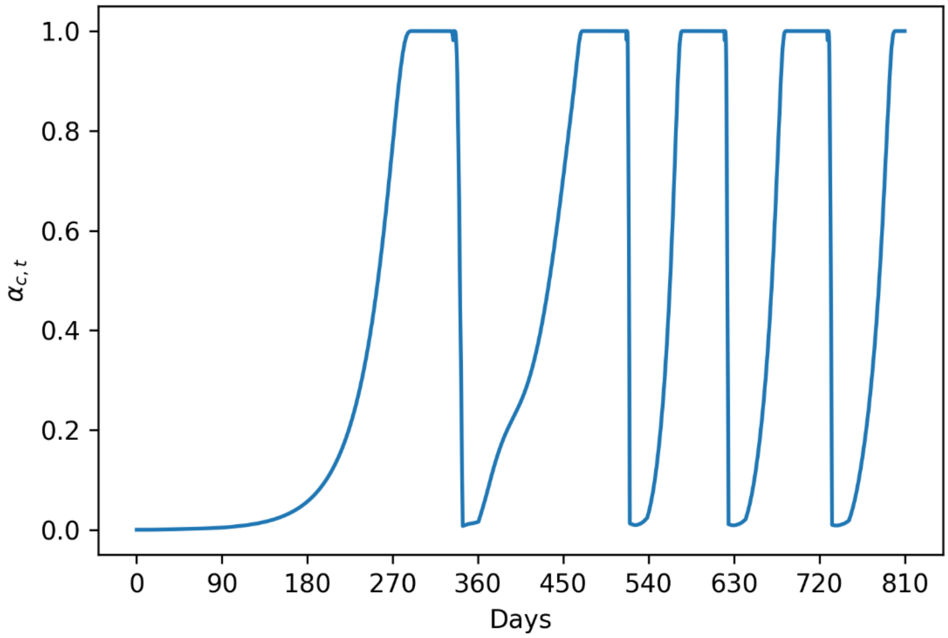

Figure 10. Probability of no treatment for severe infection-Hard lockdown.

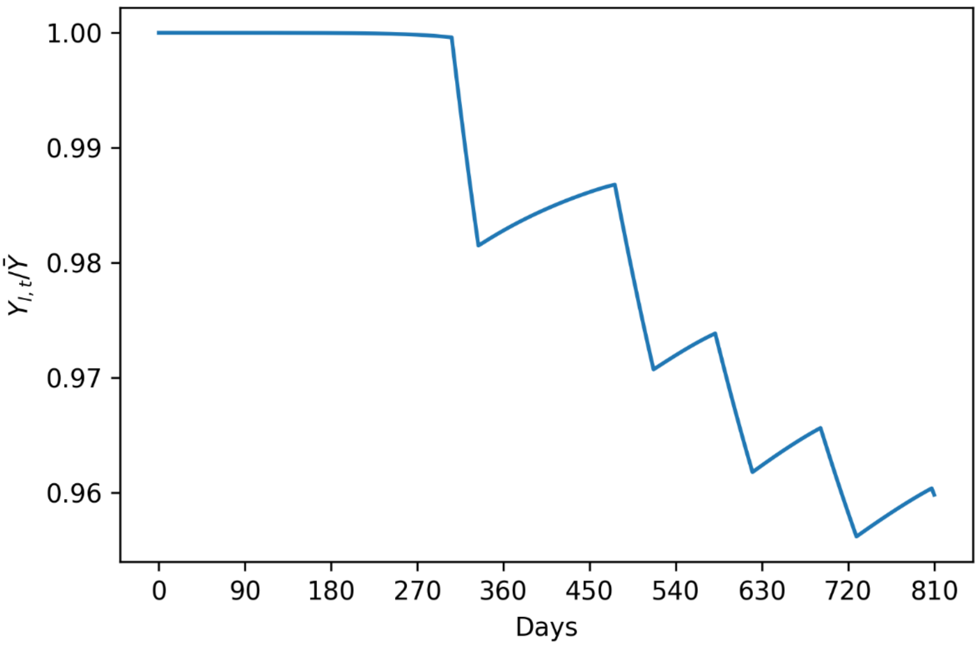

Figure 11. Actual to full employment output-Hard lockdown.

The probability of no treatment for both types of patients show different pattern in this case compared to the case of no lockdown. It is evident from Figure 9 and 10 that probability of no treatment for the mild case never reaches unity but the same for the severe case remains close to unity most of the time. This happens because availability of hospital beds has stronger role as a constraining factor than the availability of doctors. This has happened in other countries also including the developed countries with much better health infrastructure compared to the developing world. Once the lockdown is imposed, α_m,t and $\alpha_{c,t}$ start falling. The policy of lockdown helps reduce number of infections as well as death but it continues in the form of a pandemic for longer time. However, the contraction of the GDP is around 5% and 6% until eight and nine quarters respectively which are higher than in the no lockdown case. A comparison with the policy of no lockdown case highlights the issue of lives versus livelihood. With no lockdown death is much higher after about one and half years when the disease reaches steady state but the contraction in the GDP is lower and the economy has an early recovery. On the other hand a policy of hard lockdown reduces infection and death in current times but extends the period of pandemic and loss of output is much higher.

Now we consider the case of soft lockdown. In the previous section we proposed three cases of soft lockdown based on policy rule pertaining to the constraints on the availability of doctors and hospital infrastructure, viz. linear, convex and concave, assuming that the implementation started at some level of stress in the health system in comparison to that of the hard lockdown. Tables 4 and 5 provide the simulation results for convex and concave rules respectively, assuming it was initiated at 75% stress level of the health system. The results are also presented in Figure 12-16 for the convex case of adjustment and Figure 17-21 for the concave case. Both the cases show that the soft lockdown policy lies intermediate between the policy of no lockdown and the policy of hard lockdown in terms of levels of infection and case fatality. However, infection and fatality are higher in the convex case than in the concave case. The fall in GDP is lower in the convex case than in the concave case. This happens because of the fact that in the concave rule the extent of lockdown is stringent than in the convex rule, that is to say rate of change in the extent of lockdown is proportionately higher when the constraint binds under the concave rule and vice-versa for the convex rule. In other words the θ_t value is lower in the former than in the latter. The probability values of no treatment for both mild and severe infections follow similar pattern though somewhat higher in the convex rule. In fact the probability of no treatment for the severe case shows persistence at unit value while in the concave rule it changes frequently. In the case of linear policy rule the lockdown is imposed at the same rate as the change in the extent of the constraint, and is intermediate between these two cases. It is not shown here.

Table 5. Evolution of the disease: Soft lockdown with convex lockdown rule (m=10).

Day	$A_0+\sum\Delta A$	$\sum\Delta I_m$	$\sum\Delta I_c$	R_T	R_0	D	$D/(D+Ro)$	$D_h/(D_h+R_h)$	$Y_t/\bar{Y}$
90	0.068578	0.020557	0.001581	0.045884	0.01505	0.000009	0.00059765	0.00042	0.999997
180	0.908478	0.278246	0.02221	0.627009	0.20964	0.000163	0.00077775	0.00304	0.999979
270	10.931416	3.381367	0.314639	7.609496	2.537447	0.007974	0.00313268	0.00103	0.999826
360	66.742692	25.195253	5.373748	59.745885	21.953006	0.338950	0.01520504	0.00523	0.993306
450	130.242584	48.736130	10.544657	116.226099	43.156517	0.835690	0.01899632	0.00684	0.993438
540	200.652374	78.294375	18.704967	191.915135	74.508186	1.747277	0.02291347	0.00869	0.982745
630	255.618242	101.158125	24.628613	248.017861	97.663702	2.301442	0.02302245	0.00865	0.977938
720	312.730417	123.578814	29.838048	303.080311	119.095162	2.815248	0.02309276	0.00859	0.977245
810	378.590477	149.050012	35.972030	363.943986	141.814143	3.267405	0.02252116	0.00840	0.979131

Note: (i) The level variables are in Million. (ii) Lockdown imposed (lifted) on 298th (379th), 403th (544th), 564th (611th), 643th (715th), 739th (NA) days. (iii) Active cases as on 810th day is 3.968464 million and the fatality rate for the whole period (upto 810th day) is 2.2521%.

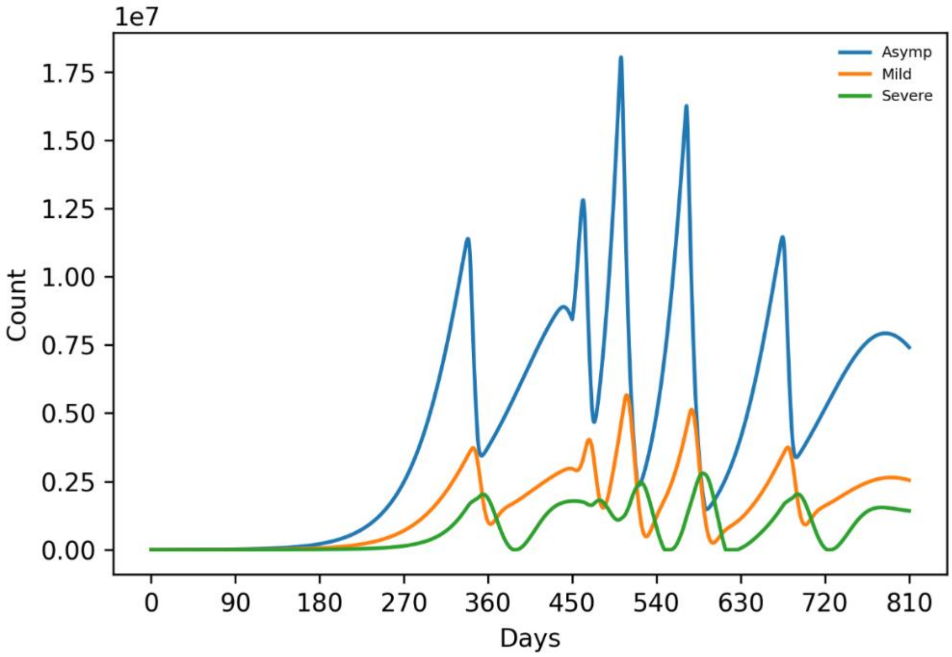

Figure 12. Evolution of the disease - Soft lockdown with convex adjustment.

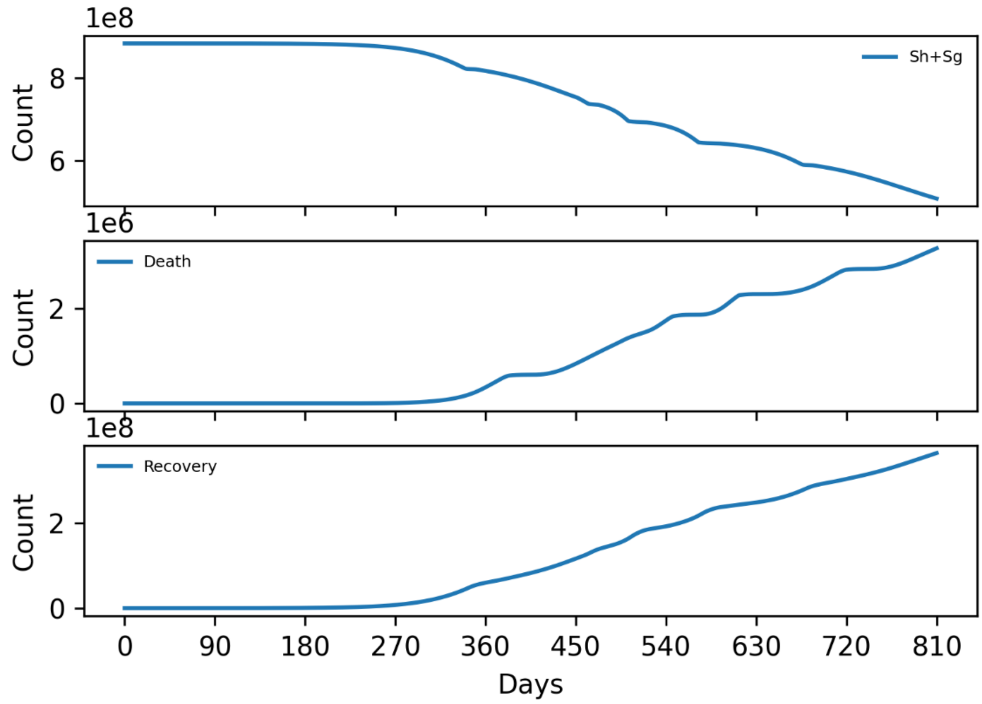

Figure 13. Susceptible, Death and Recovery-Soft lockdown with convex adjustment.

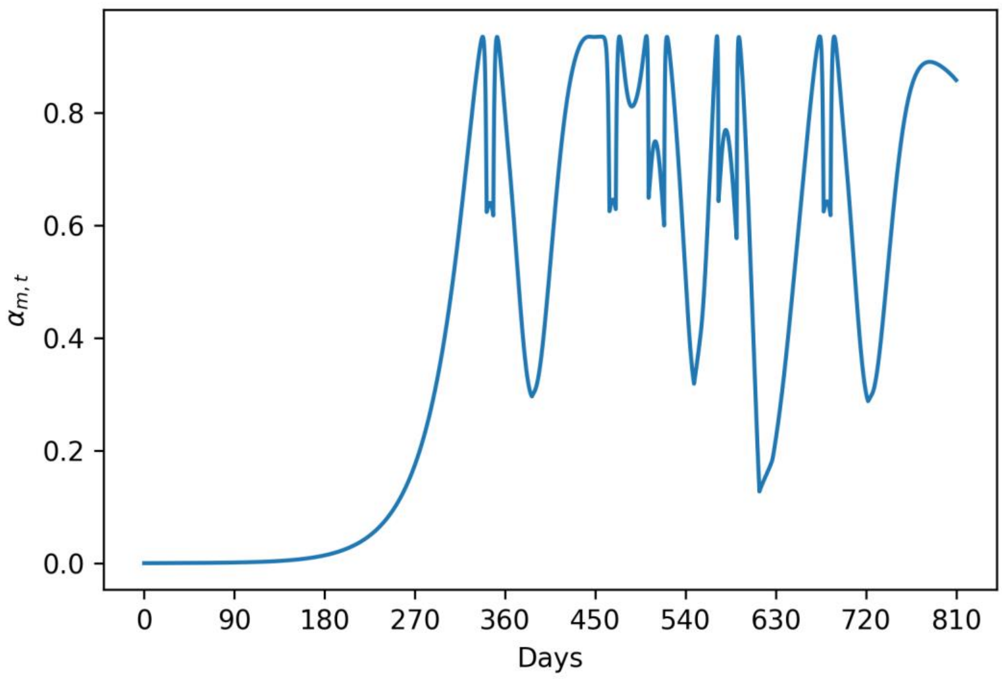

Figure 14. Probability of no treatment for mild infection-Soft lockdown with convex adjustment.

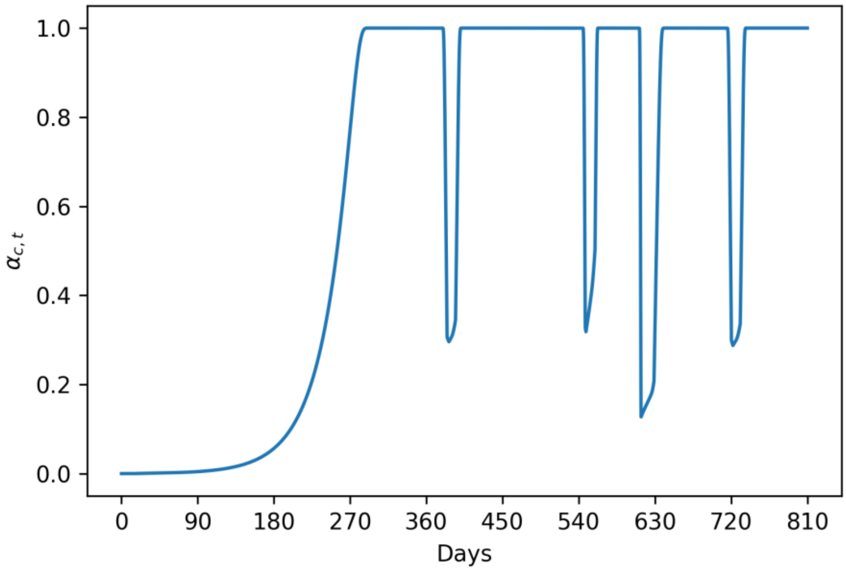

Figure 15. Probability of no treatment for severe infections-Soft lockdown with convex adjustment.

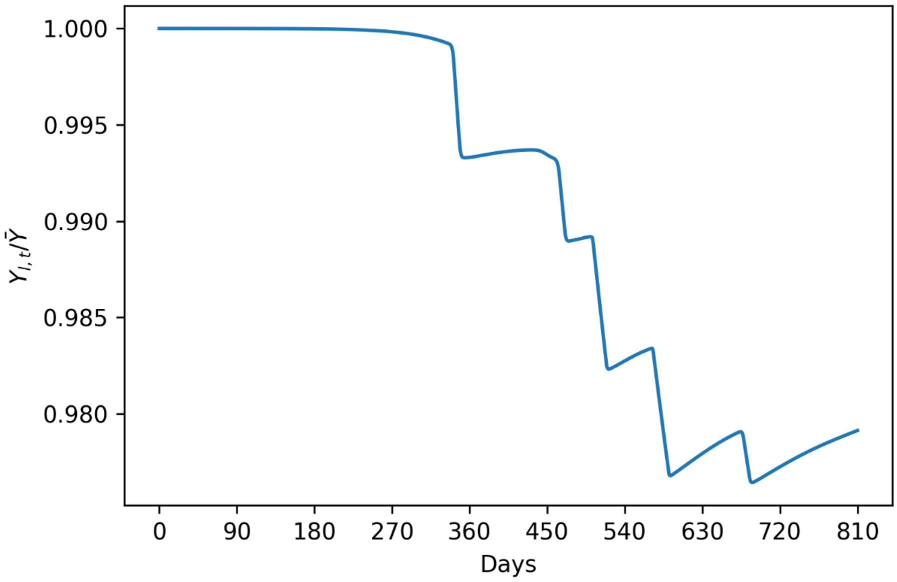

Figure 16. Actual to full employment output-Soft lockdown with convex adjustment.

Table 6. Evolution of the disease: Soft lockdown with concave lockdown rule (m=0.1).

Day	$A_0+\sum\Delta A$	$\sum\Delta I_m$	$\sum\Delta I_c$	R_T	R_o	D	$D/(D+R_o)$	$D_h/(D_h+R_h)$	$Yt/\bar{Y}$
90	0.068578	0.020557	0.001581	0.045884	0.015050	0.000009	0.0005976	0.00042	0.999997
180	0.908478	0.278246	0.022210	0.627009	0.209641	0.000163	0.0007769	0.00304	0.9999787
270	10.931416	3.381367	0.314639	7.609496	2.537447	0.007974	0.0031327	0.00103	0.9998258
360	54.909014	20.46236	3.747231	49.23810	18.54456	0.311984	0.0165451	0.00573	0.9935718
450	98.726333	37.98049	7.257810	92.90803	35.93729	0.675336	0.0184454	0.00676	0.9906948
540	151.91089	57.92799	11.70361	140.8163	54.84736	1.029413	0.0184229	0.00661	0.9818690
630	207.79640	80.78329	16.34023	193.9818	74.31376	1.345205	0.0177798	0.00619	0.9754746
720	246.34204	98.61818	20.85618	240.0414	94.14243	1.705955	0.0177985	0.00610	0.9702436
810	287.69567	114.9017	24.45994	281.2060	111.1979	2.089437	0.0184437	0.00630	0.9703420

Note: (i) The level variables are in Million. (ii) Lockdown imposed (lifted) on 298th (364th), 390th (446th), 471th (515th), 545th (591th), 617th (662th), 692th (740th), 768th (NA) days. (iii) Active cases as on 810th day is 1.61436 million and the fatality rate for the whole period (upto 810th day) is 1.84437%. (iv) Death as % of population on 630th (810th) day is 0.152% (0.236%). (v) Herd immunity as % of population on 630th (810th) day is 23.49% (32.52%).

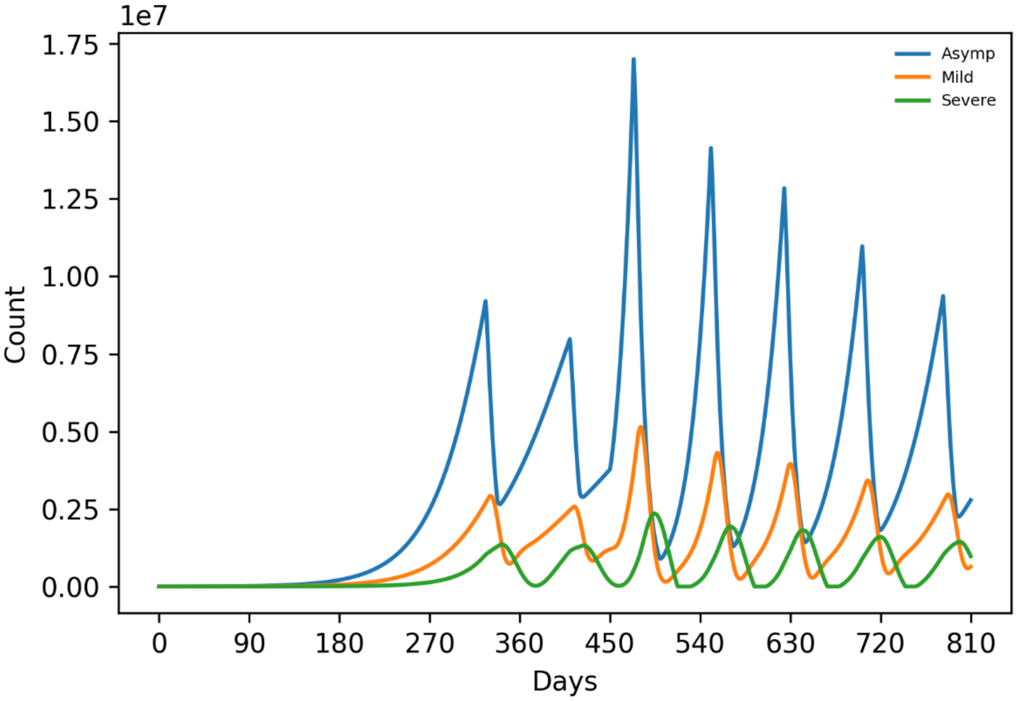

Figure 17. Evolution of the disease-Soft lockdown with concave adjustment.

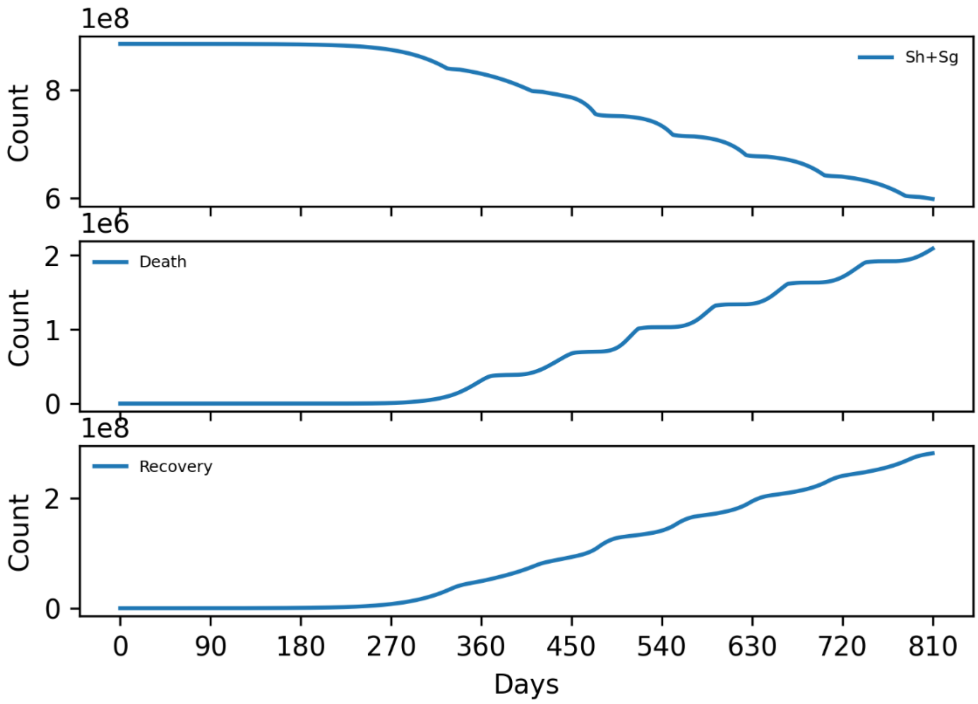

Figure 18. Susceptible, death & recovery-Soft lockdown with concave adjustment.

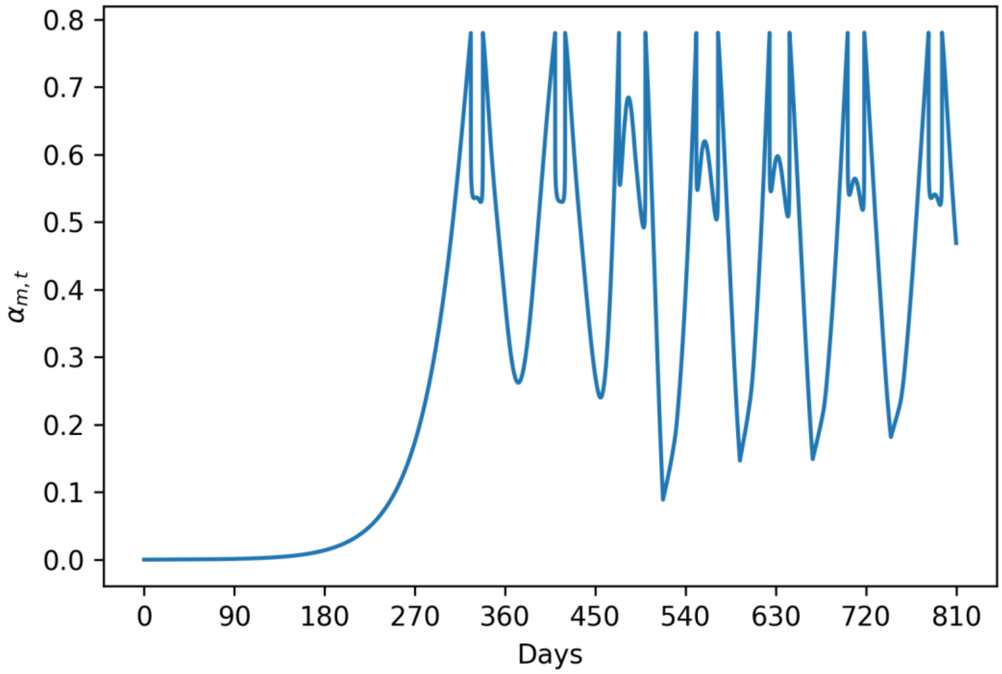

Figure 19. Probability of no treatment for mild infection-Soft lockdown with concave adjustment.

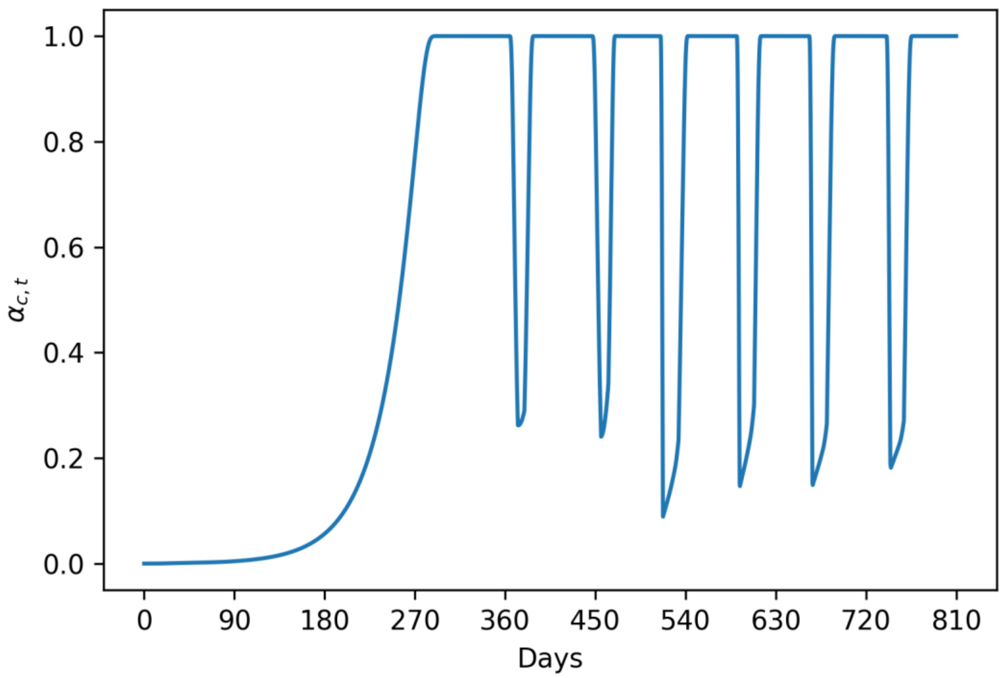

Figure 20. Probability of no treatment for severe infection-Soft lockdown with concave adjustment.

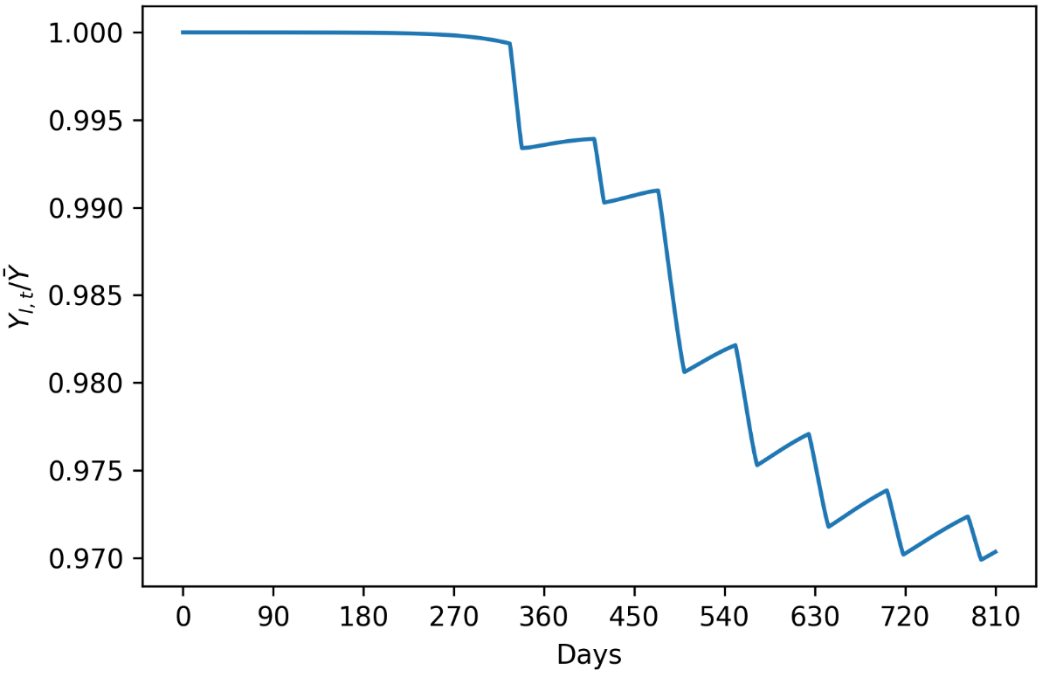

Figure 21. Actual to full employment output-Soft lockdown with concave adjustment.

The above policy rules give options to the policy makers for choosing between lives versus livelihood. The policy of no lockdown has lower adverse impact on income and employment but a higher fatality in the current times while the opposite happens for the policy of hard lockdown. The policy of soft lockdown has actually been followed in many countries, especially with a popular government in power across the globe, because it does not contract income and employment as much as for the hard lockdown rule but has lower fatality in the current times than in the no lockdown case. This choice problem assumes its importance in the absence of a cure in the form of vaccine or even if a vaccine is available, its efficacy in the face of mutation of the virus. It is upto the policy makers to decide about the lives versus livelihood question and accordingly takes decision about the implementation of the lockdown policy.

Finally, we provide an evaluation of the policy regimes in terms of minimization of the loss function Ψ_t over the period of analysis, viz. 810 days. Among the various forms of the loss function based on the power m, the quadratic form is used generally in economic analyses. It is minimised with respect to the two policy variables, viz. μ and θ. The numerical optimisation is undertaken for values of μ from 1/100 to 100 at discrete intervals with multiplication factor of 10 while that for θ from 0.3 to 0.7 with a difference of 0.1. Table 7 and Figure 22 describe this case. It is evident that the minimum is obtained at μ=100, θ=0.7. The value of μ=100 corresponds to the case of convex adjustment as the case load tightens the constraint on health infrastructure while a value of θ=0.7 implies a very high degree of soft lockdown, closer to no lockdown. As a matter of fact the minimum of the loss function for the no lockdown case (i.e. θ= 1) with μ=100 obtains at 2.83e+20 which is lower than what is obtained for θ=0.7 and μ=100 which is 1.53E+21. Thus when the policy makers follow a quadratic loss function for policy choice, a convex adjustment rule for tightening or relaxing the health facilities as and when the health constraint binds or not and a policy of less stringent lockdown give the optimal choice.

Table 7. Loss Function Ψ_t for m=2.

↓ m θ →	0.3	0.4	0.5	0.6	0.7
1/100	1.53E+22	1.06E+22	7.15E+21	4.62E+21	2.73E+21
1/10	1.30E+22	9.34E+21	6.44E+21	4.23E+21	2.55E+21
1	7.62E+21	5.68E+21	4.11E+21	2.83E+21	1.79E+21
10	9.47E+21	6.06E+21	4.16E+21	2.90E+21	1.94E+21
100	1.12E+22	7.09E+21	4.41E+21	2.64E+21	1.53E+21

Note: The values are in current million USD. (ii) Minimum obtained at m=100, θ=0.7.

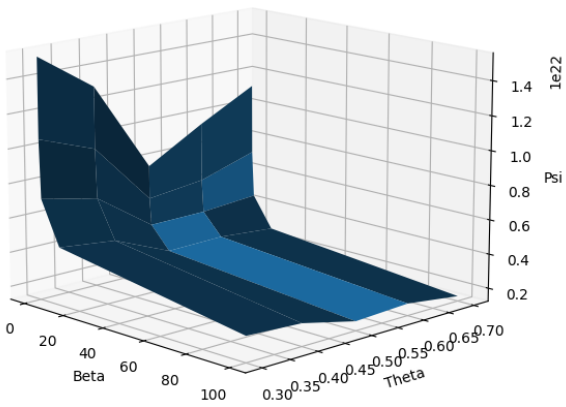

Figure 22. Loss Function Ψ_t for m=2.

The case of linear loss function is provided in Table 8 and Figure 23. In this case, however, the minimum is achieved for θ in the interval (0.4 0.5) for μ=1/100. This combination implies a more stringent lockdown with a concave adjustment rule when the constraint on the health facilities binds. The minimum of the loss function for the hard lockdown case with μ=1/100 is obtained at 1.1437e+12 which corresponds to θ =0.5. The results in the two cases vary not only in terms of numerical values but also by the policy regime of lockdown. As m is raised the effect of loss of GDP outweighs more than the cost of death. Hence as m rises, say for from m=1 to m =2 implying a quadratic loss function in the deviation of actual from the target values of output and death favours no lockdown. On the other hand a higher value of life, χ may give higher importance to life, but unlikely to favour a policy of no lockdown or soft lockdown in this model. Thus it is evident that the policy regime is very much dependent on the type of loss function of the policy makers. It is not only a question of lives versus livelihood, but how the loss of lives versus livelihood is looked upon. This is reflected in policy of almost zero restriction against Covid-19 in most of the Asian countries in the recent times as reported in The Economist, Oct. 09, 2021.

Table 8. Loss Function Ψ_t for m=1.

↓ m θ →	0.3	0.4	0.5	0.6	0.7
1/100	2.23E+12	2.22E+12	2.22E+12	2.23E+12	2.24E+12
1/10	2.23E+12	2.23E+12	2.24E+12	2.26E+12	2.28E+12
1	2.56E+12	2.58E+12	2.61E+12	2.65E+12	2.69E+12
10	3.26E+12	3.34E+12	3.34E+12	3.36E+12	3.35E+12
100	3.27E+12	3.35E+12	3.43E+12	3.59E+12	3.72E+12

Note: The values are in current million USD. (ii) Minimum obtained at m=1/100, 0.4≤θ≤ 0.5.

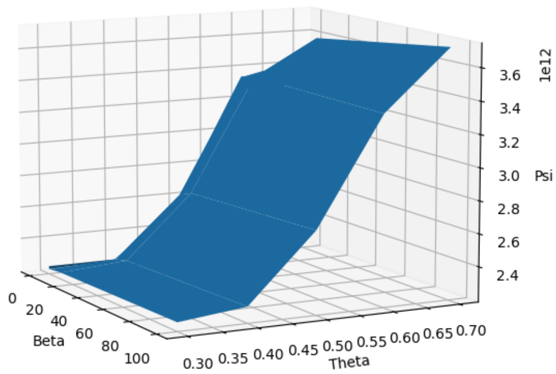

Figure 23. Loss Function Ψ_t for m=1.

The economic implication of the a sudden outbreak of an epidemic, especially for which no cure is immediately available is introduced in terms of the issue of lives versus livelihood. This is introduced minimising a loss function which can be thought of as a welfare function under different policy paradigms. The policy of no lockdown has lower adverse impact on income and employment but a higher fatality in the current times while the opposite happens for the policy of hard lockdown. Some variant of the policy of soft lockdown has been followed in many countries, especially with a popular government in power across the globe, because it does not contract income and employment as much as for the hard lockdown rule but has lower fatality in the current times than in the no lockdown case. This choice problem assumes its importance in the absence of a cure in the form of vaccine or even if a vaccine is available, its efficacy in the face of mutation of the virus. It is upto the policy makers to decide about the lives versus livelihood question and accordingly takes decision about the implementation of the lockdown policy. However, the outcome on the lives and livelihood is not independent of the type of loss function of the policy makers. It is not only a question of lives versus livelihood alone but how the loss of lives versus livelihood is looked upon in terms of the objective of the policy makers.

4. Conclusion

Pandemic like Covid-19 threw everyone, from scientists to politicians, common people, and government worldwide in the same footing. To find out ways to stop/reduce rate of infection and to reduce fatalities are need of the hour. Since the disease had no known cure by medication, only way to let people treat symptomatically to relieve and let heal (with or without other related severe illnesses/side effects) or die. Barring certain age group (especially children and young adults) since the disease does not discriminate between poor and rich, or nationality or special sects, health system of all the countries in the world stretched beyond their limit to address issues during the peak of the infection. Only measures that could be adopted to ease the health system and not fall apart is to reduce the mixing of the people. Hence the policy of lockdown and other restrictions, such as quarantine are advocated to reduce contacts across population. But all these measures adversely affect economic activities worldwide, perhaps much more than any other crisis before.

Our aim in this paper has been exploring, whether there is a way to minimise the effect of the lockdown on the economy, whether there is any optimal way to set the restrictions to reduce mixing, depending on the situation of each country, contingent on the specific economic structure and the relative value of human life. We classified the interventions into three groups, viz. no lockdown, hard lockdown and soft lockdown.

The paper has provided alternative policy options between lives versus livelihood under different lockdown regimes. It is further shown that the question of lives versus livelihood is contingent on how the policy makers put penalty when the objective fails to reach the target-a quadratic loss function gives optimal choice of no lockdown with a convex adjustment rule for constrained health services while a linear loss function achieves the optima with a policy of hard lockdown with concave adjustment rule for constrained health facilities. It is further shown that the countries where relative values of human life is higher a harder form of the lockdown to reduce the stress in the health system is optimal and attain the optimal level in balancing the economic activities, whereas the countries where the value of human life is relatively lower would be better off with softer version of the lockdown, that is allowing certain level of economic activities varying according to the stress on the health system and/or infection and fatality rate. Thus the economy would be functioning relatively at a higher level of activities than with a policy of hard lockdown. As a result people with low level of earnings, generally daily wage earners, would not be adversely affected, especially when the government did not compensate then when unemployed. This is especially true in low income countries with no effective way of unemployment registration and large informal/ unorganized sector.

We have restricted our analysis to one mutation of the virus (or say, 2nd wave, due to mutation), one can do similar exercise for third or more waves. Although, mutation had been introduced here exogenously keeping the fatality rate same, it can be introduced as an endogenous random phenomenon with lower fatality rate, as it has happened in several countries. Similarly, repeated infection has not been introduced in this paper (even after mutation), but it can easily be incorporated and enhance the chances of the 2nd and more waves and more slow rate of going to stability. Under the model conditions, stability is reached faster in the no lockdown case (630 days) but with more deaths in the current times. Hard lockdown achieves stability at a later date (more than 2200 days) with lower fatality, but reduces level of output as well. The policy of soft lockdown with a loss function linear in the deviation of output from target and number of deaths can help countries opt for suitable way to run the economic activities at the same time relieving the stress on the health system. The resultant outcomes of fatality, aggregate output and steady state of the disease spread lie somewhere in between the policies of hard lockdown and no lockdown.

Purpose of this model is not meant for forecasting; rather it is an exploratory model for understanding the spread and arrest of the pandemic like Covid-19 over time. The model is simulated with the parameter values that may resemble a country like India. With suitable amendments it can be used to study the characteristics of the disease elsewhere. Further, this paper has explored only homogenous single working population though differentiated in one respect, viz. a large general population and a small population group of health workers. One can incorporate demographic composition with an elderly age group and a group of children in this model to make it a model for all groups of population. However, qualitative results will still hold, with children group having lower infection and lower fatality rates and elderly having higher infection and fatality rates and proportion of them in the population varying from country to country.

The model structure can easily be extended in a number of directions. One can bring in interaction with other (one or more) population groups across regions or countries through trade/travel (i.e., mixing between districts, between states/provinces as happened in Europe, India and the USA and elsewhere across the globe). Such an interactive model will allow the policy- makers to study the local characteristic for any regions keeping interaction with neighbouring regions and help implement more effective measures/decisions. We discussed the effect of the availability of a vaccine as the inverse of mutation. However, one can extend the paper with an explicit treatment of the introduction of the vaccine at any point of time or Ł with varying success rates at different points in time for the study of the eventual stability of vaccination. One can incorporate expectation of the discovery of vaccine at the first period and optimise the loss function with the expectation of the discovery.

Overall this is only an exploratory model, which would allow several other component to make it a more realistic framework for the policymakers to device a decision-making mechanisms.

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Conflict of interest

All the authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

References

Acemoglu, D., Chernozhukov, V., Werning, I. & Whinston, M. D. (2021). A Multi-Risk SIR Model with Optimally Targeted Lockdown. *American Economic Review: Insights*, forthcoming. https://doi.org/10.1257/aeri.20200590

Agarwal, M., Kanitkar, M., Phillip, D., Hajra, T., Singh, A., Singh, A., Singh, P.P. & Vidyasagar M. (2022). SUTRA: An Approach to Modelling Pandemics with Undetected Patients, and Applications to COVID-19. arXiv.org/abs/2101.09158v6. doi.org/10.48550/arXiv.2101.09158.

Al-Qahtani, M., AlAli, S., Rahman, A. K. A., Alsayyad, A. S., Otoom, S. & Atkin, S. L. (2021). The prevalence of asymptomatic and symptomatic COVID19 disease in a cohort of quarantined subjects. *Journal of Infectious Diseases*,102, 285-288. https://doi.org/10.1016/j.ijid.2020.10.091.

Alvarez, F. E., Argente, D. & Lippi F. (2021). A Simple Planning Problem for Covid-19 Lockdown. *American Economic Review: Insights* 3, 367-82. https:// doi.org/10.1257/aeri.20200201

Anastassopoulou, C., Russo, L., Tsars, A. & Siettos C. (2020). Data-based analysis, modelling and forecasting of the COVID-19 outbreak. *PLoS ONE* 15(3), e0230405. https://doi.org/10.1371/journal.pone.0230405.

Aspri, A., Beretta, E., Gandolfi, A. & Wasmer, E. (2021). Mortality containment vs. Economics Opening: Optimal policies in a SEIARD model. *Journal of Mathematical Economics* 93: 102490. https://doi.org/10.1016/j.jmateco.2021.102490.

Barro, R. J., Urs'ua, J. F. & Weng, J. (2020). The coronavirus and the great influenza pandemic: Lessons from the "Spanish flu" for the coronavirus's potential effects on mortality and economic activity. (NBER Working Paper No. 26866). https://doi.org/10.3386/w26866.

Basak, G. K., Chakraborty, C., Das, P. K. (2021): Optimal Lockdown Strategy in a Pandemic: An Exploratory Analysis for Covid-19. (arXiv.org pre-print). https://arxiv.org/abs/2109.02512.

Berger, D. W., Herkenhoff, K. F. & Mongey S. (2020). An seir infectious disease model with testing and conditional quarantine. (NBER Working Paper No. 26901). https://doi.org/10.3386/w26901.

Bertozzi, A. L., Franco, E., Mohler, G., Short, M. B. & Sledge, D. (2020). The challenges of modeling and forecasting the spread of COVID-19. (arXiv.org). https://arxiv.org/abs/2004.04741.

Bethune, Z. A. & Korinek, A. (2020). Covid-19 infection externalities: Trading off lives vs. livelihood. (NBER Working Paper No. 27009). https://doi.org/10.3386/w27009.

Bhattacharjee, S., Liao, S., Paul, D., & Chaudhuri, S. (2022): Inference on the dynamics of COVID-19 in the United States, *Sci Rep* 12, 2253. https://doi.org/10.1038/s41598-021-04494-z.

Border, A., Gray, D., Islam, A. & Bhuiyan, S. (2021). A literature review of the economics of COVID-19. *Journal of Economic Surveys* 35, 1007-1044. https://doi.org/10.1111/joes.12423.

Born, B., Dietrich, A. & Mu¨ller, G. J. (2021). Do Lockdowns Work? A Counterfactual for Sweden. *PLoS ONE* 16(4): e0249732. https://doi.org/10.1371/journal.pone.0249732.

Bryant, P. & Elofsson, A. (2020). Estimating the impact of mobility patterns on COVID-19 infection rates in 11 European countries. (medRxiv.org). https://doi.org/10.1101/2020.04.13.20063644.

Buonsenso, D., De Rose, C., Pierantoni, L. (2021). Doctors' shortage in adults COVID-19 units: a call for pediatricians, *European Journal of Pediatrics* 180, 2315–2318. https://doi.org/10.1007/s00431-021-03995-3.

Casseli, F., Grigori, F., Lian, W. & Sandri, D. (2021). Protecting Lives and Livelihoods with Early and Tight Lockdowns. *THE BE Journal of Macroeconomics*, *2*, 241-268. https://doi.org/10.1515 bejm-2020-0266.

Caulkins, J., Grass, D., Feichtingelear, G., Hartl, R., Kort, P. M., Prskawetz, A., Seidi, A. & Warzaczek, S. (2020). How long should the COVID-19 lockdown continue? *PLoS ONE 15* 12: e0243413. https://doi.org/10.1371/journal.pone.0243413.

Caulkins, J. P., Grass, D., Feichtinger G, Hartl RF, Kort PM, Prskawetz A, Seidl A, Wrzaczek S. (2021). The optimal lockdown intensity for covid-19. *Journal of Mathematical Economics* 93: 102489. https://doi.org/10.1016/j.jmateco.2021.102489.

Chowell, G., Hyman, J. M., Bettencourt, L. M. & Castillo-Chavez, C. (2009). Mathematical and statistical estimation approaches in epidemiology. *Springer*. https://doi.org/10.1007/978-90-481-2313-1.

Dimdore-Miles, O. & Miles, D. (2020). Assessing the spread of the novel coronavirus in the absence of mass testing. *International Journal of Clinical Practice*. https://doi.org/10.1111/ijcp.

Eichenbaum, M. S., Rebelo, S. & Trabandt, M. (2020). The Macroeconomics of Epidemics. (NBER Working Paper No. 26882). https://doi.org/10.3386/w26882.

Farboodi, M., Jarosch, G. & Shimer, R. (2020). Internal and External Effects of Social Distancing in a Pandemic. (NBER Working Paper No. 27059). https://doi.org/10.3386/w27059.

Fosco, C. & Zurita, F. (2021). Assessing the short-run effects of lockdown policies on eco- nomic activity, with an application to the Santiago Metropolitan Region, Chile. *PLoS ONE* 16, 6: e0252938. https://doi.org/10.1371/journal.pone.0252938.

Gatto, M., Bertuzzo, E., Mari, L., Miccoli, S., Carraro, L., Casagrandi, R. & Rinaldo A. (2020). Spread and dynamics of the COVID-19 epidemic in Italy: Effects of emergency containment measures. *Procedings of National Academy of Sciences* 117, 19: 10484–10491. https://doi.org/10.1073/pnas.2004978117.

Gollier, C. (2020). Cost-benefit analysis of age-specific deconfinement strategies. *Journal of Public Economic Theory* 22, 1746-7. https://doi.org/10.1111/jpet.12486

Gori, L., Manfredi, P., Marsiglio, S. & Sodini, M. (2021). COVID-19 epidemic and mitigation policies: Positive and normative analyses in a neoclassical growth model. *Journal of Public Economic Theory*, 1–25. https://doi.org/10.1111/jpet.12549.

Government of India. (2020): Provisional Estimates of Annual National Income 2019-2020 and Quarterly Estimates of Gross Domestic Product for the Fourth Quarter (Q4) of 2019-20. National Statistical Office, Ministry of Statistics and Programme Implementation.

Gregory, V., Menzio, G. & Wiczer, D. (2020). Pandemic Recession: L or V-Shaped? Quarterly Review, Federal Reserve Bank of Minneapolis 40, 1, 1-40. https://doi.org/10.21034/qr.4011

Grigorieva, E., Khailov, E. & Korobeinikov, A. (2020). Optimal quarantine strategies for COVID-19 control models. (arXiv.org). https://arxiv.org/abs/2004.10614.

Guerrieri, V., Lorenzoni, G., Straub, L. & Werning I. (2021). Macroeconomic Implications of COVID 19: Can Negative Supply Shocks Cause Demand Shortages? *American Economic Review*. https://www.aeaweb.org/articles?id=10.1257/aer.20201063.

Hall, R. E., Jones, C. I. & Klenow, P. J. (2020). Trading Off Consumption and Covid-19 Deaths. (Stanford University Working Paper No. 20-026). http://siepr.stanford.edu/sites/default/files/publications/20-026.pdf.

Hethcote, H. W. (2000). The Mathematics of Infectious Diseases. *SIAM Review*,42,4,599-653. http://www.siam.org/journals/sirev/42-4/37190.htnil.

ILO. (2018). India Wage report: Wage policies for decent work and inclusive growth. ILO, Geneva.

Ji, Y., Ma, Z., Peppelenbosch, M. P., & Pan, Q. (2020). Potential association between COVID-19 mortality and health-care resource availability, *The Lancet* 8, 4. https://doi.org/10.1016/S2214-109X(20)30068-1.

Jones, C. J., Philippon, T. & Venkateswaran, V. (2020). Optimal Mitigation Policies in a Pandemic: Social Distancing and Working from Home. (NBER Working Paper No. 26984). https://doi:10.3386/w26984.

Kaplan, G., Moll, B. & Violante, G. (2020). The Great Lockdown and the Big Stimulus: Tracing the Pandemic Possibility Frontier of the US. (BFI Working Paper No. 2020-119). http://bfi.uchicago.edu/working-paper/2020-119.

Kermack, W. O., McKendrick & A. G. (1927). A contribution to the mathematical theory of epidemics. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 115, 772, 700–721. https://www.jstor.org/stable/94815

Majumder, A. & Madheswaran, S. (2018). Value of Statistical Life in India: A Hedonic Wage Approach. (ISEC Working Paper No. 407). http://www.isec.ac.in/-Agamoni Majumder and Madheswaran-Final.pdf.

Moolla, I., Hiilamo, H. (2023). Health system characteristics and COVID-19 performance in high-income countries. *BMC Health Services Research*, 23, 244. https://doi.org/10.1186/s12913-023-09206-z.

Rampini, A. (2020). Sequential Lifting of COVID-19 Interventions with Population Heterogeneity. (NBER Working Paper No. 27063). https://doi:10.3386/w27063.

Robeznieks, A. (2022). What we've learned about COVID-19, burnout and the doctor shortage, Association of American Medical Colleges. https://www.ama- assn.org/print/pdf/node/85241.

Sameni, R. (2020). Mathematical Modeling of Epidemic Diseases; A Case Study of the COVID-19 Coronavirus. (arXiv.org Pre-print). https://arxiv.org/abs/2003.11371.

Schelling, T. C. (1968). The Life You Save May Be Your Own. In S. B. Chase Jr. (Ed.). Problems in Public Expenditure Analysis (pp. 127-62). Washington D.C.: Brookings Institution. http://hdl.handle.net/10822/762904.

Sharma, D., Bouchaud, J.-P., Gualdi, S., Tarzia, M. & Zamponi, F. (2021). V–, U–, L– or W–shaped economic recovery after Covid-19: Insights from an Agent Based Model. *PLoS ONE* 16, 3: e0247823. https://doi.org/10.1371/journal.pone.0247823.

The Print. (2021). 1 doctor for 1,511 people, 1 nurse for 670—Covid exposes India's health- care 'fault lines', Feb. 11, https://theprint.in/health/1-doctor-for-1511-people-1-nurse-for-670-covid-exposes-indias-healthcare-fault-lines/602784.

Viscusi, W. K. & Aldy, J. E. (2003). The Value of a Statistical Life: A Critical Review of Market Estimates throughout the World. *The Journal of Risk and Uncertainty* 27, 1, 5-76. https://www.jstor.org/stable/41761102.