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Optimizing Labor Allocation based on Multiobjective Decision Making Using Improved Hungarian Algorithm

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ABSTRACT

The aim of this study is to solve the problem of labor allocation in cases where the company has fewer employees than the number of existing jobs based on the evaluation of work quality according to multiple objectives. Optimizing labor allocation not only benefits the company by maximizing the use of human resources, but also saves employees' energy. In addition, employees are assigned to tasks that match their skills and qualifications, maximizing their productivity. The research results show that the multi-objective decision-making algorithm based on the Hungarian algorithm is a suitable method to help leaders of companies solve the aforementioned problem text.

KEYWORDS

Human Resource Management; Hungarian algorithm; Multiobjective Decision Making Model; Optimization

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1. Introduction

The correct choice of human resource management (HRM) policy is crucial for the efficient activity and competitiveness of an organization in the knowledge-based economy [Mathis et al (2014) and Spencer (2008)]. The labor market changes rapidly due to globalization and technological advancements, which require the development of new conceptual approaches and scientifically substantiated methods in HRM policy. HRM is a specialized form of managerial activity that focuses on managing the human factor, including their competencies, motivational principles, and professional abilities, to support the organization's activity strategy [Armstrong (2009)]. To make adequate decisions in personnel planning, selection, recruitment, retention, promotion, development, training, and motivation, decision-makers must evaluate a wide range of information about employees, compare applicants based on multiple heterogeneous attributes, and consider multiple influences, preferences, interests, and possible consequences. These HRM tasks are multi-objective, and decision-makers must consider the volume, quantitative and qualitative nature, complexity, and inconsistency of the information flow, which makes them semi-structured tasks. Objective models are either impossible or extremely difficult to construct. In addition to the problems encountered during the generation and selection of managerial decisions, decision-makers' preferences and experts' competence, such as knowledge, intuition, and experience, should also be considered.

One of the core issues to be addressed in HRM is labor assignment. Proper and optimized labor assignment brings many benefits to both the business and the employees. For employees, they will have the following advantages: 1) Increased opportunities for self-improvement, skill enhancement, and professional development; 2) Clear understanding of the tasks to be performed, ensuring complete fulfillment of assigned responsibilities; 3) Reduced pressure and workload intensity, and elimination of unsuitable work tasks; 4) Efficient and timely completion of tasks. For businesses, appropriate labor assignment helps increase professionalism in management, gain employee trust, minimize usage costs, and maximize employee potential. However, if labor assignment is based solely on the leader's intuition, it will be difficult to avoid mistakes in management and consequently, fail to achieve the aforementioned benefits. Therefore, to allocate employees in suitable positions, businesses need to quantify employees' abilities to complete a specific task to make accurate decisions and lay a foundation for digitizing labor assignment, making human resource management increasingly scientific. Furthermore, when considering the current capacity of each employee, businesses always rely on various different evaluation criteria, with different weights for each criterion. This helps comprehensively evaluate the employee's ability, but also makes it difficult to make decisions in labor assignment. Therefore, the authors see the need to quantify the results of performance evaluations based on multiple objectives, to develop a suitable formula that represents the specific value of an employee's capacity, to optimize labor allocation.

References from the papers of Greshilov (2014), Mammadova (1997), Mikoni (2015), Zongmin et al (2017) provide the fundamental principles of multi-criteria decision-making and the application of intelligent systems to support decision-making in HRM. One of the primary challenges in multi-criteria decision-making tasks is acquiring information and determining its nature, type, and processing methods. Additionally, it is crucial to determine the number of alternatives and their descriptive attributes, the hierarchal structure, and the presentation of expert knowledge. In particular, the handling of linguistic variables in HRM tasks requires the selection of appropriate methods based on fuzzy set and fuzzy logic theories [Kofman (1982) and Zadeh (1965)]. To address these challenges, methodological approaches based on intelligent technologies, methods, and computer systems of decision-making support need to be developed [Trakhtenherts (2001)].

The objective of this research includes two main contents:

(1) Develop a methodological approach for managerial decision-making in human resource management (HRM) tasks, specifically in labor allocation when the number of employees is less than the number of tasks. The input data

in this case is based on performance evaluation results, which have specific features such as multi-objectivity and heterogeneity of data, quantitative criteria, ambiguity, the need for considering expert evaluation of their weight, and the influence of experts' competence on the decision-making process.

(2) Propose an improved mathematical method for optimizing labor allocation in the above situation.

With the aforementioned objectives, the structure of the paper is described as follows: Firstly, the authors present relevant studies on labor allocation based on performance evaluation and previous research on applying various methods in labor allocation optimization. Next, the paper discusses the assumptions in the specific situation, particularly focusing on addressing the allocation issue when the number of workers is less than the number of tasks, providing the mathematical formulation of the multi-objective optimization problem. Subsequently, the authors present the Hungarian algorithm and propose an improved version of the Hungarian algorithm to solve the stated multi-objective optimization problem. Finally, the paper applies the proposed improved Hungarian method as an illustrative example based on the initial assumptions.

2. Literature review

2.1. Concepts related to the process of assigning tasks and evaluating productivity, performance

In general, making decisions on assigning tasks to the workforce as well as evaluating productivity and performance has been developed from many different theories and scientific papers. Weiss & Hartle (1997) argued that performance management is a process of establishing a common understanding of what needs to be achieved and an appropriate approach for managing individuals in order to achieve success. In other words, with the aim of completing and improving the quality, managers must have a clear understanding of expected results based on objectives, then plan, organize, and control work assignments strictly in the workplace.

According to Baron & Armstrong (1998), productivity and performance management play an important role when developing the ability to work in a team, as well as of the employees who make individual contributions to the goals. It can be seen as a process that needs to be done regularly in order to make a general assessment and focus on future tasks. In general, the management of productivity, performance, and operational efficiency in an organization is covered in various forms such as budgeting, planning and organizing effective assignments, and management by objectives (MBO). Thus, understanding the goals, strengths, and weaknesses as well as the organizational structure will be the key point for the assignment policies and evaluation.

Brewster et al (2018) explained that performance management is an idea that approaches the problem of structural model performance, in which tasks must be performed focusing on goals, monitoring, appraising, developing, and rewarding human resources in order to increase motivation. In addition, the cohesion between the team in particular and the whole organization generally will also be greatly increased. It is obvious that the approach broadened the manager's vision of measuring effectiveness carefully. In fact, there are many different ways in the formula for calculating results and employee performance including management by goals (MBO), Behavioral Fixed Scale (BARS), 360-degree feedback, key performance indicators (KPIs), and many other formats.

In the research, Armstrong (2009) illustrated that the overall purpose of performance management is to develop people's capacity to meet the expected standards and exploit their full potential. Thanks to the evaluation of performance, self-development can be increased. Therefore, managers should assign tasks which suit the capacity of each individual, or group in an organization.

In conclusion, the perspectives and theories mentioned above demonstrate a general understanding of the issue of improving organizational performance through the analysis and assigning tasks to the right individuals at right time. It can be seen that with the dense amount of work, it is important for managers to assign responsibilities and tasks to members of a team appropriately based on standards and assessments, which will help ensure goals are achieved as expected, always on time, and on budget.

2.2. Research Overview

Baeva et al (2009) demonstrated the problem of assigning tasks as well as responsibilities for subordinates since it must be suitable for their knowledge, and skills. By collecting data from the tests for employees, then analyzing through a mathematical model, including the knap-snack problem and linear assignment problem, to optimize human resource efficiency.

Astuti et al. (2018) in the research proved that the incompatibility between workload and working capacity is one of the challenges of organizations when they want to achieve optimal efficiency. The author has designed a question model related to workload and calculation methods with the goal of evaluating performance, thereby determining the needs of office workers on task assignments applied to State-Owned Enterprise PT.X.

In addition, many organizations, especially colleges, and universities, are going to use emerging technology in human resource management in order to adapt to the rapid development of the economy. In general, Haiyan Tu (2019) described the process of collecting accumulated data, analyzing human resource situations automatically, extracting valuable information from large amounts of complex data to make decisions, and supporting human resource management.

The article by Rashid K. et al. (2020) discusses the popularity of modular construction technology in the United States due to its many advantages over traditional construction methods. As modular construction factories operate as an assembly line, the number of workers at various workstations affects the efficiency of overall production. To optimize resource allocation in uncertain work environments at modular construction factories, the article introduces a tool framework combining discrete event simulation and genetic algorithm to support data-driven decision making. A real-world study has shown that optimizing the assignment of available workers can reduce production time by up to 15%. This demonstrates the potential of the proposed method as a practical tool to optimize resource allocation in uncertain work environments at modular construction factories.

Wang et al. (2022) applied the AHP method to calculate the weights of four key indicators, including material reward, the performance bonus plan, the employee capacity development training, and the employee job promotion system, to build a human resource management model, strategy by optimizing these key factors. In addition, the other objective of the research is to define the current problem of an IoT enterprise, which is human resources management is not able to satisfy the Internet industry, and restricts development. Thus, employee allocation for the right positions and tasks affects IoT organizations.

The study by Wang & Zhang (2022) develops an optimal human resource allocation model based on PSO, the methods of system, and quantitative evaluation. The author believes that the distribution of human resources has a direct effect on the efficiency of human resource utilization in enterprises, which in turn has a strong impact on profits and labor productivity. From the mathematical modeling method, they optimize the personnel configuration and provide a specific quantitative management method for performance analysis, system analysis, and structure allocation.

The authors named Li Liu and Seechen Yong (2022) analyzed the actual needs, business processes, and existing problems of enterprises in performance management. In particular, with the multi-objective decision-making mathematical model, the author's team can assist administrators in optimal task arrangement. That is to say, the research aims to combine the evaluation of work efficiency and standards to have the best decision-making for orienting responsibilities and tasks. It can be said that without standard performance evaluation systems, it will be difficult for optimizing human resource management to achieve the expected goals.

Yi Li and Wang Linna (2022) have developed a model for project scheduling that takes into account constraints related to human resources and emphasizes differences in competency. One of the key benefits of this model is its use of indicators that provide an objective and rational evaluation of personnel competency, while maintaining a rigorous and scientific approach. By transforming the complex double-objective problem of minimizing the total construction period and total cost into a single-objective maximization problem with a comprehensive index, the model can be optimized using a genetic algorithm. As a result, this approach provides an effective solution to project scheduling problems that are resource-constrained and require the consideration of personnel competency.

3. Research hypothesis

Based on the importance of performance management and task allocation based on the quality evaluation of task execution, the authors are interested in situations where human resources are allocated appropriately and achieve maximum effectiveness based on this evaluation. The traditional Hungarian algorithm allows the optimization of *N* people completing *N* tasks, where each task is performed by only one person. This is based on the efficiency of person *i* in performing task *j* and is denoted as P_{ij}. The optimal arrangement is found to maximize task efficiency. However, in real-world scenarios, task quality is often constrained by many conditions, such as working time, task efficiency, and task risks… Therefore, the use of the multiobjective decision-making (MDCM) method is entirely appropriate to help solve the optimal arrangement problem in the situation presented above. There are two different situations in MDCM: one where there are more people than tasks, and the other where there are more tasks than people. The optimal arrangement when there are more people than tasks has been presented in the study by Li Liu and Seechen Yong (2022). Therefore, the goal of this paper is to present an optimal solution for the remaining problem, which is to arrange employees in the most optimal way for the case where the number of employees is less than the number of current tasks.

Assuming that *M* people are arranged to complete *N* tasks where $N > M$, each person can do multiple tasks, and each task is done by one person. In addition, the personnel arrangement must satisfy *K* specified objectives. With *K* objectives, the measure of how well the *k*-th objective of task *n* is achieved when person *m* does it is P_{mn}^{k}, where $m = 1,2,\ldots,M; n = 1,2,\ldots,N; k = 1,2,\ldots,K$. A mathematical model is constructed to meet the optimal assignment requirements, expressed in mathematical equations as follows:

$$\max F_k = \sum_{m=1}^{M}\sum_{n=1}^{N} p_{mn}^{k}\mathrm{e}_{mn}\,; k = 1,2,\ldots,K, \tag{1}$$

under the following conditions:

$$\begin{cases} \sum_{n=1}^{N} \mathrm{e}_{mn} = 1 \\ \sum_{n=1}^{N}\sum_{m=1}^{M} e_{mn} = N \\ \mathrm{e}_{mn} = 0,1. \end{cases} \tag{2}$$

in which $e_{mn} = 1$ means assigning person *m* to task *n*; conversely, $\mathrm{e}_{mn} = 0$. In case the objective with the measure of achievement has a higher value, it is better, the maximum value of determines the highest quality and efficiency of the task. In case the objective has a lower value, it is better, the lower value determines that the time taken to complete the task is shorter and more efficient. An illustrative example will be given for the situation corresponding to the above mathematical model. The article presents a specific situation as follows: suppose there are $M = 4$ employees who need to be $N = 5$ assigned tasks, and $K = 3$, with corresponding objectives of time, quality of work, and risk level.

4. Methodology

4.1. The Hungarian algorithm

The Hungarian algorithm, also known as the Kuhn-Munkres Algorithm, is a combinatorial optimization algorithm that solves the assignment problem in polynomial time. It was developed by two mathematicians, Harold Kuhn, and James Munkres, in the early 1950s. The assignment problem involves finding the optimal assignment of *n* tasks to *n* agents, where each agent can perform only one task and each task can be performed by only one agent. The objective is to minimize the total cost of the assignment, which is the sum of the costs of the individual assignments. The Hungarian Algorithm works by constructing a matrix of costs and finding a matching that minimizes the total cost. The algorithm starts by finding the smallest element in each row and subtracting it from all the elements in the row. It then finds the smallest element in each column and subtracts it from all the elements in the column. This creates a new matrix with some zero elements, which represent possible assignments. The algorithm then tries to find a matching by selecting a zero element and marking it as part of the matching. It then marks all the elements in the same row and column as the selected zero element as unavailable. If there is a row or column with only one available zero element, the algorithm marks it as part of the matching and continues. If not, it repeats the process of selecting a zero element until a matching is found. The Hungarian Algorithm has a worst-case time complexity of O (n^3), which makes it efficient for solving large assignment problems. It has found applications in a wide range of fields, including economics, engineering, and computer science. The flowchart of the algorithm is illustrated in Figure 1.

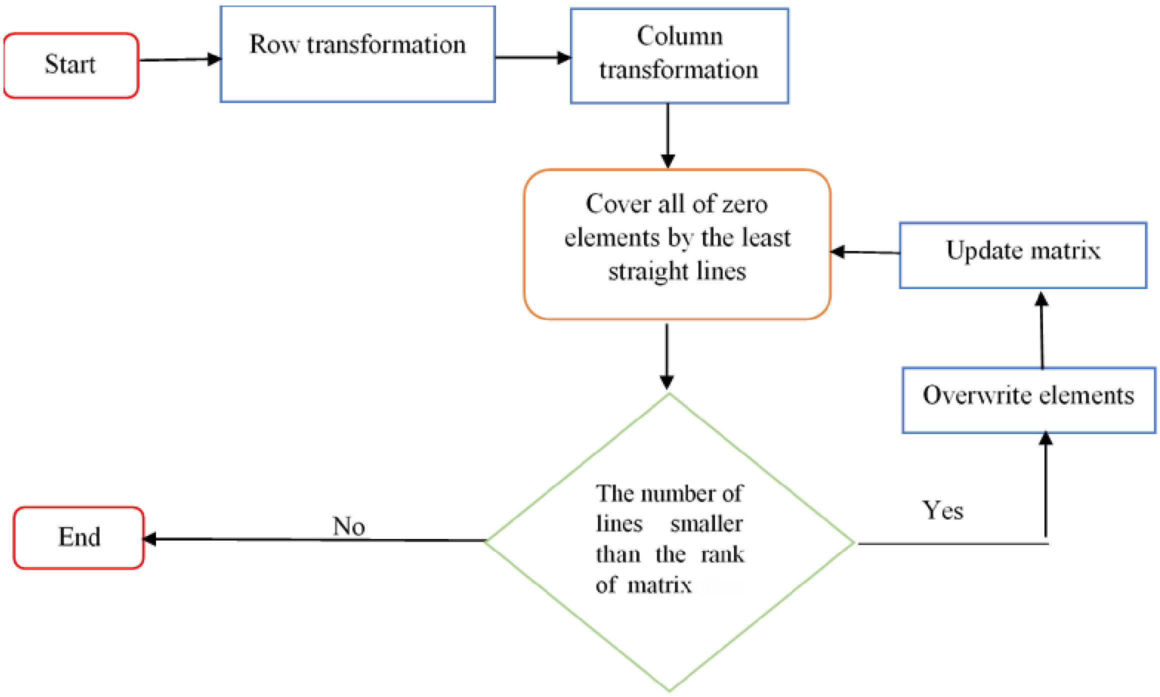

Figure 1. Flowchart of the Hungarian algorithm.

To better understand the traditional Hungarian algorithm, let's consider the following problem: "Assuming a cost matrix $C = [c_{ij}]$ with dimensions $m \times m$ and an assignment matrix $X = [x_{ij}]$ with the same dimensions, we can define c_{ij} as the cost of assigning worker *i* to task *j*, and x_{ij} as a binary variable that equals 1 if worker *i* is assigned to task *j*, and 0 otherwise. The minimum cost assignment problem can then be formulated as a linear programming problem using the following mathematical expression:

$$\underset{x}{argmin}\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij}x_{ij}\,. \tag{3}$$

And X satisfies $\sum_{i=1}^{m}x_{ij}=1;\sum_{j=1}^{m}x_{ij}=1;x_{ij}\in\{0,1\}$."

If the problem to be solved requires finding the maximum cost, it can be transformed into a minimum cost problem as follows:

$$\underset{x}{argmin}\sum_{i=1}^{m}\sum_{j=1}^{m}-c_{ij}x_{ij} \tag{4}$$

The solution to the above problem provides us with the steps of the Hungarian algorithm. The detailed solution of the problem can be found in the research of Kuhn (1955). The steps of the Hungarian algorithm can be described as follows:

Step 1: Row reduction. For each row, subtract the smallest element of that row from all elements in that row.

Step 2: Column reduction. For each column, subtract the smallest element of that column from all elements in that column.

Step 3: Draw lines through zeros with the minimum number of lines. If the number of lines is less than the matrix dimension, proceed to step 4. If the number of lines is equal to or greater than the matrix dimension, go to step 5.

Step 4: Find the smallest element not covered by the lines drawn in step 3. Subtract this value from all uncovered elements, add it to all elements at the intersection of the drawn lines, and return to step 3.

Step 5: Determine the optimal solution and assign tasks to employees.

First, the zeros in the matrix need to be identified. We start with the first row or column. If there is only one zero in a row or column, then assign a (1) to that element, which indicates that the corresponding task is assigned to the corresponding employee. If there are multiple zeros in a row or column, we prioritize assigning a (1) to the first zero seen from left to right that has not yet been marked. Whenever a (1) is assigned to a zero in a row (column), other zeros in the same column (row) are marked as (2), indicating that this task can be done by other employees. This process is repeated until all zeros in the coefficient matrix are marked as either (1) or (2).

We have a simple example to illustrate the use of the Hungarian algorithm. Let's say a company has 5 jobs T_1, T_2, T_3, T_4 and T_5 needs to assign them to 5 employees E_1, E_2, E_3, E_4 and $E_5$to complete a project. The wages the company needs to pay for completing the tasks are described in Table 1. The problem to be solved is how to arrange the jobs to get the lowest cost. The problem-solving process will use the Hungarian algorithm as follows:

Table 1. The specific cost for each employee by task.

Employee/Task	T_1	T_2	T_3	T_4	T_5
E_1	3	4	7	2	9
E_2	4	8	5	3	4
E_3	2	5	6	7	4
E_4	3	6	9	3	5
E_5	5	2	8	7	3

Then, we have the following matrix:

$$\begin{bmatrix} 3 & 4 & 7 & 2 & 9 \\ 4 & 8 & 5 & 3 & 5 \\ 2 & 5 & 6 & 7 & 4 \\ 3 & 6 & 9 & 3 & 5 \\ 5 & 2 & 8 & 7 & 3 \end{bmatrix} \tag{5}$$

Applying the Hungarian algorithm to the above matrix, we obtain the following transformed matrices through the algorithm steps:

Table 2. The steps of the Hungarian algorithm through the illustrative example.

Step	Matrix
1	$\begin{bmatrix} 1 & 2 & 5 & 0 & 7 \\ 1 & 5 & 2 & 0 & 2 \\ 0 & 3 & 4 & 5 & 2 \\ 0 & 3 & 6 & 0 & 2 \\ 3 & 0 & 6 & 5 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} -2 \\ -3 \\ -2 \\ -3 \\ -2 \end{bmatrix}$ $[-2 - 1]$ $\downarrow$
2	$\begin{bmatrix} 2 & 3 & 4 & 1 & 0 \\ 1 & 5 & 0 & 0 & 1 \\ 0 & 3 & 2 & 5 & 2 \\ 0 & 3 & 4 & 0 & 2 \\ 3 & 0 & 4 & 5 & 1 \end{bmatrix}$
3	$\begin{bmatrix} 2 & 3 & 4 & 1 & 0 \\ 1 & 5 & 0 & 0 & 1 \\ 0 & 3 & 2 & 5 & 2 \\ 0 & 3 & 4 & 0 & 2 \\ 3 & 0 & 4 & 5 & 1 \end{bmatrix}$ The number of lines is smaller then 5. The smallest uncovered number is 1. We subtract this number from all uncovered elements and add it to all elements that are covered twice and return step 3.
4	$\begin{bmatrix} 1 & 1 & 2 & 0 & 5 \\ 2 & 5 & 0 & 1 & 1 \\ 0 & 2 & 1 & 5 & 0 \\ 0 & 2 & 3 & 0 & 0 \\ 4 & 0 & 4 & 6 & 0 \end{bmatrix}$
3	$\begin{bmatrix} 1 & 1 & 2 & 0 & 5 \\ 2 & 5 & 0 & 1 & 1 \\ 0 & 2 & 1 & 5 & 0 \\ 0 & 2 & 3 & 0 & 0 \\ 4 & 0 & 4 & 6 & 0 \end{bmatrix}$ There are 5 lines required to cover all zeros, so we perform step 5.
5	$\begin{bmatrix} 1 & 1 & 2 & 0(1) & 5 \\ 2 & 5 & 0(1) & 1 & 1 \\ 0(1) & 2 & 1 & 5 & 0(2) \\ 0(2) & 2 & 3 & 0(2) & 0(1) \\ 4 & 0(1) & 4 & 6 & 0 \end{bmatrix}$

From the final matrix in step 5, we arrange the tasks by rewriting the matrix with the convention: the position of 0 (1) in each corresponding row or column is rewritten as 1, the other positions are determined as 0, and then the optimal solution of the problem is obtained. The optimal solution is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

From the above matrix, we can see that task T_5 is assigned to E_4, task T_3 is assigned to E_2, task T_1 is assigned to E_3, task T_4 is assigned to E_1, and task T_2 is assigned to E_5. This can be seen from the following table:

Table 3. Task assignment table to achieve optimal cost requirements.

Employee/ Task	T_1	T_2	T_3	T_4	T_5
E_1				x	
E_2			x		
E_3	x				
E_4					x
E_5		x			

At that time, the total cost to be paid is: $2 + 5 + 2 + 5 + 2 = 16$, and this is also the minimum cost.

4.2. The improved Hungarian algorithm

Optimizing labor allocation based on quantifying performance evaluation on multiple criteria with different weights should rely on the following:

(1) Consider the suitability of employees for a task based on multiple criteria and goals.

(2) Provide evaluation levels on the suitability of employees for each criterion or goal.

(3) Quantify these levels using an appropriate scale.

(4) Evaluate the importance of each criterion to determine the corresponding weight.

(5) Develop a general formula for performance evaluation that represents the performance metric of the employee when performing that task.

(6) Apply a suitable optimization algorithm to achieve the optimal goal of labor allocation.

In the hypothetical research scenario where labor allocation is based on performance evaluation results, which in turn are based on multiple objectives, optimizing labor allocation requires combining with the multi-objective decision-making method. Therefore, this research improves the traditional Hungarian algorithm to solve the situation presented in the hypothetical study. The idea of the improvement is that each objective has a different weight in the overall evaluation, so creating a cost matrix will depend on the weight and performance evaluation results through averaging. The steps to solve the problem are presented as follows:

Step 1. Complete the fuzzy multiobjective aggregation matrix.

Assuming P_{mn}^{k} represents the contribution value of the nth task to the kth objective performed by the mth person, where $n \in [1, N], m \in [1, M]$ and $k \in [1, K]$. Then, we can establish the contribution matrix $P_k = (P_{mn}^{k})_{M \times N}$. Based on the contribution matrix P_k, the fuzzy relation matrix is determined and denoted as $U_k = (u_{mn}^{k})_{M \times N}$. Equation (3) represents that a larger value indicates a better achievement of the target, while equation (4) represents that a smaller value indicates a better achievement of the target:

$$u_{mn}^{k} = \frac{p_{mn}^{k} - p_{kmin}}{p_{kmax} - p_{kmin}} \quad (7)$$

$$u_{mn}^{k} = \frac{p_{kmax} - p_{mn}^{k}}{p_{kmax} - p_{kmin}} \quad (8)$$

The two formulas (7), (8) have a few points that need to be noted:

- Let p_{kmax} represent the highest value in matrix P_k, while p_{kmin} represents the lowest value in matrix P_k. By applying the given formula, the matrix P_k undergoes a transformation to become the fuzzy relation matrix $U_k = (u_{mn}^k)_{M\times N}$ for the respective *k*-th target. Simultaneously, the weight vector is established through the assessment conducted by the managers.
- In practical scenarios, leaders take into account the significance of tasks and evaluate the importance of quality, time constraints, and risk levels associated with these tasks. When carrying out these tasks, they consider various factors. For instance, if a task is relatively intricate and holds significant importance, the emphasis would be on ensuring high-quality results, even if it requires more time to complete. This aligns perfectly with the principles outlined in the aforementioned model, where the weight vectors prioritize the quality of task completion over time requirements. Conversely, if a task is simple and of lesser importance, the requirements for the weight vector would naturally be adjusted accordingly. In such cases, the weight vector design would prioritize different factors. Assuming the target weight $W = (w_1, w_2, .., w_K)$ is provided by the leader and satisfies the equations presented above, we can express u_{mn} as the sum of the values $w_k u_{mn}^k$, where *k* ranges from 1 to *K*:

$$u_{mn} = \sum_{k=1}^{K} w_k u_{mn}^k \tag{9}$$

Here, u_{mn} represents the average value calculated based on the weight vector and corresponding values u_{mn}^k from the comprehensive evaluation of the performance of each individual's performance in achieving *K* targets. Using this method, we have the fuzzy multiobjective aggregation matrix as follows:

$$U = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1N} \\ u_{21} & u_{22} & \dots & u_{2N} \\ \dots & \dots & \dots & \dots \\ u_{M1} & u_{M2} & \dots & u_{MN} \end{bmatrix} = (u_{mn})_{M\times N} \tag{10}$$

① u_{mn} represents the fuzzy comprehensive evaluation value of *m*th person overall job performance when doing *n*th task. The higher u_{mn} indicates greater efficiency, thereby determining a fuzzy multiobjective aggregation matrix from evaluating the performance of each opjective.

② The method for determining the fuzzy multiobjective aggregation matrix of employees is described as follows: Firstly, the contribution value of the *n*th task performed by the *m*th person determined under the *k*th goal is u_{mn}^k, and this value is calculated based on equation (7) or (8). These values need to be based on the assessment of the professional competence and work ability of the employee during the work process. For each goal, the comprehensive quality assessment of the employee is converted into a 100-point scale, with an absolute score of 100. In reality, the evaluation may use different score scales with different levels of scores, in which case the value calculation can be understood as the degree of task completion calculated by probability. Secondly, the weight vector $W = (w_1, w_2, \dots, w_k)$ is given according to the requirements of the leader for that task. Thirdly, the measurement value of the relative fuzzy evaluation level of the employee when completing tasks under the above goals can be calculated according to equations (9). Finally, we have the matrix of fuzzy multiobjective aggregation obtained by (10).

Step 2. Applying the improved Hungarian algorithm to solve multi-objective assignment problems.

When the number of tasks is greater than the number of people $N > M$, it means that each person has to do multiple tasks. Assuming that each task has been assigned to one person, then among the remaining $N - M$ tasks, each task can be assigned to one of the *M* people. Therefore, the number of unassigned tasks is $N - M$. To convert the problem back to the original problem, we assume that the company has additional $M(N-M)$ employees. Now, to match the number of tasks with the number of employees, we need to add $M(N-M+1)-N=(N-M)(M-1)$ virtual tasks that have zero work efficiency when any person is assigned to them. By adding these virtual tasks, the problem still satisfies the condition that each task is performed by one and only one person, satisfying the requirements of the traditional problem. Therefore, the extended fuzzy multiobjective aggregation matrix can be constructed as follows:

$$A=(\alpha_{mn})_{M(N-M+1)\times M(N-M+1)}=\begin{bmatrix} U & 0 \\ U & 0 \\ \vdots & \vdots \\ U & 0 \end{bmatrix} \quad (11)$$

Column 1 in the above matrix has $(N-M+1)$ matrix U. The number 0 in column 2 represent the zero matrix, each these matrices have Mrows and $(N-M)(M-1)$columns. The optimal solution to the traditional assignment problem corresponding to the extended fuzzy multiobjective aggregation matrix A transformed appropriately to perform the Hungarian algorithm. The result obtained after executing the algorithm can be determined by the elements equal to 1 in the first n columns of the resulting matrix $R=(\beta_{mn})_{Z\times Z}$ with $Z=M(N-M+1)$in step 5 of Hungarian algorithm.

5. The result of implementing the improved Hungarian algorithm on the research hypothesis

Assuming that a company has 4 employees performing 5 tasks. The number of people is $M=4$, and the number of tasks is $N=5$. Task completion has 3 targets, thus $K=3$. Three objectives are completing the task with high quality and short completion time, with the lowest level of risk. Suppose that for each objective, the maximum score that can be achieved is 100 points. For the risk level objective, the higher the score, the lower the risk level the person guarantees. We categorize the evaluation levels as shown in Table 4.

Table 4. Table of work performance evaluation levels.

Score	Degree
90 – 100	Excellent
80 – 90	Good
70 – 80	Quite good
60 – 70	Average
Under 60	Below average

The initial data is determined through the above process. To facilitate the computation, the evaluation results of employees are rounded to 2 decimal places. The contribution matrices are given in (12):

$$P_1=\begin{bmatrix} 80 & 70 & 75 & 90 & 85 \\ 80 & 70 & 65 & 80 & 90 \\ 90 & 80 & 90 & 85 & 70 \\ 60 & 70 & 75 & 95 & 65 \end{bmatrix}; P_2=\begin{bmatrix} 70 & 60 & 80 & 80 & 90 \\ 60 & 50 & 80 & 90 & 65 \\ 80 & 85 & 75 & 60 & 70 \\ 95 & 60 & 65 & 70 & 60 \end{bmatrix}; P_3=\begin{bmatrix} 75 & 80 & 80 & 85 & 95 \\ 70 & 50 & 85 & 95 & 70 \\ 70 & 85 & 85 & 60 & 90 \\ 85 & 100 & 75 & 70 & 90 \end{bmatrix} \quad (12)$$

Here, P_1 is the matrix of task completion quality, P_2 is the matrix of time to complete tasks, and P_3 is the matrix measuring the level of safety in performing tasks. In matrices P_1 and P_3, larger values are better. Equation (3) is used to calculate the fuzzy relation matrix. For the time matrix P_2, smaller values are better, and equation (4) is used to calculate the fuzzy relation matrix. The calculated results are represented by matrices U_1, U_2 and U_3 respectively.

$$U_1 = \begin{bmatrix} 0.57 & 0.29 & 0.43 & 0.86 & 0.71 \\ 0.57 & 0.29 & 0.14 & 0.57 & 0.86 \\ 0.86 & 0.57 & 0.86 & 0.71 & 0.29 \\ 0.00 & 0.29 & 0.43 & 1.00 & 0.14 \end{bmatrix} \tag{13}$$

$$U_2 = \begin{bmatrix} 0.56 & 0.78 & 0.33 & 0.33 & 0.11 \\ 0.78 & 1.00 & 0.33 & 0.11 & 0.67 \\ 0.33 & 0.22 & 0.44 & 0.78 & 0.56 \\ 0.00 & 0.78 & 0.67 & 0.56 & 0.78 \end{bmatrix} \tag{14}$$

$$U_3 = \begin{bmatrix} 0.50 & 0.60 & 0.60 & 0.70 & 0.90 \\ 0.40 & 0.00 & 0.70 & 0.90 & 0.40 \\ 0.40 & 0.70 & 0.70 & 0.20 & 0.80 \\ 0.70 & 1.00 & 0.50 & 0.40 & 0.80 \end{bmatrix} \tag{15}$$

In this example, we focus on the quality of work as the primary goal and the completion time as the second objective, with the lowest level of job risk being the third goal. Assuming the manager assigns weights of the three objectives as follows:$W = (0.6,0.3,0.1)$, the result of calculating the fuzzy relation matrix is represented by matrix U in (16).

$$U = \begin{bmatrix} 0.56 & 0.46 & 0.42 & 0.68 & 0.55 \\ 0.62 & 0.47 & 0.26 & 0.47 & 0.75 \\ 0.65 & 0.48 & 0.72 & 0.68 & 0.42 \\ 0.07 & 0.50 & 0.51 & 0.81 & 0.40 \end{bmatrix} \tag{16}$$

The extended fuzzy multiobjective aggregation matrix, constructed based on the fuzzy relation matrix, is represented as matrix A in (17).

$$A = \begin{bmatrix} 0.56 & 0.46 & 0.42 & 0.68 & 0.55 & 0 & 0 & 0 \\ 0.62 & 0.47 & 0.26 & 0.47 & 0.75 & 0 & 0 & 0 \\ 0.65 & 0.48 & 0.72 & 0.68 & 0.42 & 0 & 0 & 0 \\ 0.07 & 0.50 & 0.51 & 0.81 & 0.40 & 0 & 0 & 0 \\ 0.56 & 0.46 & 0.42 & 0.68 & 0.55 & 0 & 0 & 0 \\ 0.62 & 0.47 & 0.26 & 0.47 & 0.75 & 0 & 0 & 0 \\ 0.65 & 0.48 & 0.72 & 0.68 & 0.42 & 0 & 0 & 0 \\ 0.07 & 0.50 & 0.51 & 0.81 & 0.40 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

We transform the matrix by subtracting all elements of matrix A from the maximum element of A to make the problem suitable for the input of the Hungarian algorithm.

$$S = \begin{bmatrix} 0.25 & 0.35 & 0.39 & 0.13 & 0.26 & 0.81 & 0.81 & 0.81 \\ 0.19 & 0.34 & 0.55 & 0.34 & 0.06 & 0.81 & 0.81 & 0.81 \\ 0.16 & 0.33 & 0.09 & 0.13 & 0.39 & 0.81 & 0.81 & 0.81 \\ 0.74 & 0.31 & 0.30 & 0.00 & 0.41 & 0.81 & 0.81 & 0.81 \\ 0.25 & 0.35 & 0.39 & 0.13 & 0.26 & 0.81 & 0.81 & 0.81 \\ 0.19 & 0.34 & 0.55 & 0.34 & 0.06 & 0.81 & 0.81 & 0.81 \\ 0.16 & 0.33 & 0.09 & 0.13 & 0.39 & 0.81 & 0.81 & 0.81 \\ 0.75 & 0.31 & 0.30 & 0.00 & 0.41 & 0.81 & 0.81 & 0.81 \end{bmatrix}, \tag{18}$$

We initiate the Hungarian algorithm with the matrix S obtained in (18). The final result of the transformed matrix, following the algorithm's steps, is presented in Table 5.

Table 5. Table containing the values of the elements in the final matrix of the transformation process.

0.06	0.04	0.27	0.13	0.26	0.00	0.00	0.00
0.00	0.03	0.43	0.34	0.06	0.00	0.00	0.00
0.00	0.05	0.00	0.16	0.42	0.03	0.03	0.03
0.55	0.00	0.18	0.00	0.00	0.00	0.00	0.00
0.06	0.04	0.27	0.00	0.00	0.00	0.00	0.00
0.00	0.03	0.43	0.00	0.00	0.00	0.00	0.00
0.00	0.05	0.00	0.03	0.03	0.03	0.03	0.03
0.55	0.00	0.18	0.00	0.00	0.00	0.00	0.00

The result obtained based on the Hungarian algorithm is represented by matrix R in (19).

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

Table 6. Table of job assignments on the extended fuzzy multiobjective aggregation matrix.

	T_1	T_2	T_3	T_4	T_5			
E_1								
E_2								
E_3								
E_4								
E_1								
E_2								
E_3								
E_4								

The assignment can be summarized briefly in table 7:

Table 7. Table of specific task assignments for the research hypothetical problem.

Employee	Task
E_1	T_4
E_2	T_5
E_3	T_1;T_3
E_4	T_2

6. Conclusion

Using the quantitative results of performance evaluations in practical work assignments has a significant impact on improving the overall quality of a company in many aspects. This study, by applying scientific mathematical models, specifically the improved Hungarian algorithm, has solved the problem of optimal labor allocation for cases where the number of tasks is greater than the number of employees. Solving this problem brings the following benefits:

Firstly, the algorithm is simple and suitable for use when assigning work in small and medium-sized companies.

Secondly, labor allocation can be more reasonably optimized, and overall work efficiency can be improved, which benefits both the company and its employees.

Thirdly, in practical situations, the number of tasks in a company is often much greater than the number of employees, and the number of employees cannot increase in proportion to the increase in the number of tasks. Therefore, this study has contributed to providing a specific, optimal solution direction for leaders in this situation.

Fourthly, arranging reasonable arrangements for employees and tasks is very important for business leaders. The paper uses a multi-objective decision-making mathematical model to optimize performance evaluation in personnel management. If this optimization method is integrated into a digital performance evaluation management system, it can improve the quality of personnel management in a company.

However, in this study, the evaluation of employee job performance is entirely based on the performance evaluation system and the quantitative results. Meanwhile, the development of personnel management also depends on many other factors, so there is still a need for significant supplementation and improvement to achieve the goal of scientific management, especially the flexibility of managers.

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Conflict of interest

All the authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

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